Statistical Language Model

Word Prediction

- Guess the next word...

  ... I notice three guys standing on the ???

- There are many sources of knowledge that could be helpful, including world knowledge.
- But we can do pretty well by simply looking at the preceding words and keeping track of simple counts.

Word Prediction

- Formalize this task using *N*-gram models.
- *N*-grams are token sequences of length *N*.
- Given N-grams counts, we can guess likely next words in a sequence.

Probabilistic Language Models

The goal: assign a probability to a sentence

- Machine Translation:
  - P(**high** winds tonite) > P(**large** winds tonite)
- Spelling Correction
  - The office is about fifteen **minuets** from my house
    - P(about fifteen **minutes** from) > P(about fifteen **minuets** from)
- Speech Recognition
  - P(I saw a van) >> P(eyes awe of an)
- + Summarization, question-answering, etc., etc.!!

Probabilistic Language Modeling

- Goal: compute the probability of a sentence or sequence of words:

$$P(W) = P(w_1,w_2,w_3,w_4,w_5 \ldots w_n)$$

- Related task: probability of an upcoming word:

$$P(w_5|w_1,w_2,w_3,w_4)$$

- A model that computes either of these:

$P(W)$ or $P(w_n|w_1,w_2 \ldots w_{n-1})$ is called a **language model**.

- Better: **the grammar** But **language model** or **LM** is standard

How to compute P(W)

- How to compute this joint probability:

P(its water is so transparent that the)=

$$\frac{\text{Count(its water is so transparent that the)}}{\text{Count(its water is so transparent that)}}$$

P(its, water, is, so, transparent, that)

- **Intuition:** let's rely on the **Chain Rule of Probability**

The Chain Rule: General

- The definition of conditional probabilities

  $P(A \mid B) = P(A, B) / P(B)$

  Rewriting: $P(A, B) = P(A \mid B) P(B)$

- More variables:

  $P(A,B,C,D) = P(A)P(B|A)P(C|A,B)P(D|A,B,C)$

- The Chain Rule in General

$P(x_1,x_2,x_3,\ldots,x_n) = P(x_1)P(x_2|x_1)P(x_3|x_1,x_2)\ldots P(x_n|x_1,\ldots,x_{n-1})$

The Chain Rule: joint probability in sentence

$$P(w_1 w_2 \ldots w_n) = \prod_i P(w_i \mid w_1 w_2 \ldots w_{i-1})$$

P("its water is so transparent") =

P(its) × P(water|its) × P(is|its water) ×

P(so|its water is) × P(transparent|its water is so)

How to estimate these probabilities

- Could we just count and divide?

$$P(\text{the} \mid \text{its water is so transparent that}) = \frac{Count(\text{its water is so transparent that the})}{Count(\text{its water is so transparent that})}$$

- No! Too many possible sentences!
- We'll never see enough data for estimating these

Markov Assumption

Andrei Markov

- Simplifying assumption:

$$P(\text{the} \mid \text{its water is so transparent that}) \approx P(\text{the} \mid \text{that})$$

- or maybe

$$P(\text{the} \mid \text{its water is so transparent that}) \approx P(\text{the} \mid \text{transparent that})$$

Markov Assumption

$$P(w_1 w_2 \ldots w_n) \approx \prod_i P(w_i \mid w_{i-k} \ldots w_{i-1})$$

- In other words, we approximate each component in the product

$$P(w_i \mid w_1 w_2 \ldots w_{i-1}) \approx P(w_i \mid w_{i-k} \ldots w_{i-1})$$

Simplest case: Unigram model

$$P(w_1 w_2 \ldots w_n) \approx \prod_i P(w_i)$$

Some automatically generated sentences from a unigram model

```
fifth, an, of, futures, the, an, incorporated, a, a,
the, inflation, most, dollars, quarter, in, is, mass

thrift, did, eighty, said, hard, 'm, july, bullish

that, or, limited, the
```

Quiz

Which is assigned higher probability by a unigram language model for English?

- P(I like ice cream)
- P(the the the the)
- P(Go to class daily)
- P(class daily go to)

Bigram model

- Condition on the previous word:

$$P(w_i \mid w_1 w_2 \ldots w_{i-1}) \approx P(w_i \mid w_{i-1})$$

texaco, rose, one, in, this, issue, is, pursuing, growth, in, a, boiler, house, said, mr., gurria, mexico, 's, motion, control, proposal, without, permission, from, five, hundred, fifty, five, yen

outside, new, car, parking, lot, of, the, agreement, reached

this, would, be, a, record, november

N-gram models

- We can extend to trigrams, 4-grams, 5-grams
- In general this is an insufficient model of language
  - because language has **long-distance dependencies**:

"The computer which I had just put into the machine room on the fifth floor crashed."

- But we can often get away with N-gram models

Estimating N-gram Probabilities

Estimating bigram probabilities

- **The Maximum Likelihood Estimate (MLE)**

$$P(w_i \mid w_{i-1}) = \frac{count(w_{i-1}, w_i)}{count(w_{i-1})}$$

$$P(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

An example

$$P(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

<s> I am Sam </s>

<s> Sam I am </s>

<s> I do not like green eggs and ham </s>

$P(\texttt{I} \mid \texttt{<s>}) = \frac{2}{3} = .67$ $P(\texttt{Sam} \mid \texttt{<s>}) = \frac{1}{3} = .33$ $P(\texttt{am} \mid \texttt{I}) = \frac{2}{3} = .67$

$P(\texttt{</s>} \mid \texttt{Sam}) = \frac{1}{2} = 0.5$ $P(\texttt{Sam} \mid \texttt{am}) = \frac{1}{2} = .5$ $P(\texttt{do} \mid \texttt{I}) = \frac{1}{3} = .33$

More examples: Berkeley Restaurant Project sentences

- can you tell me about any good cantonese restaurants close by
- mid priced thai food is what i'm looking for
- tell me about chez panisse
- can you give me a listing of the kinds of food that are available
- i'm looking for a good place to eat breakfast
- when is caffe venezia open during the day

Raw bigram counts (absolute measure)

- Out of 9222 sentences

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

Raw bigram probabilities (relative measure)

- Normalize by unigrams:

i	want	to	eat	chinese	food	lunch	spend
2533	927	2417	746	158	1093	341	278

- Result:

	i	want	to	eat	chinese	food	lunch	spend
i	0.002	0.33	0	0.0036	0	0	0	0.00079
want	0.0022	0	0.66	0.0011	0.0065	0.0065	0.0054	0.0011
to	0.00083	0	0.0017	0.28	0.00083	0	0.0025	0.087
eat	0	0	0.0027	0	0.021	0.0027	0.056	0
chinese	0.0063	0	0	0	0	0.52	0.0063	0
food	0.014	0	0.014	0	0.00092	0.0037	0	0
lunch	0.0059	0	0	0	0	0.0029	0	0
spend	0.0036	0	0.0036	0	0	0	0	0

Bigram estimates of sentence probabilities

P(<s> I want english food </s>) =

P(I|<s>)

× P(want|I)

× P(english|want)

× P(food|english)

× P(</s>|food)

= .000031

What kinds of knowledge?

- P(english|want) = .0011
- P(chinese|want) = .0065
- P(to|want) = .66
- P(eat | to) = .28
- P(food | to) = 0
- P(want | spend) = 0
- P (i | <s>) = .25

world

grammar

grammar (contingent zero)

grammar (structural zero)

The **right to food**, and its non variations, is a human right protecting the right for people to feed themselves in dignity

Practical Issues

- We do everything in log space
  - Avoid underflow: multiplying extremely small numbers
  - Adding is faster than multiplying

$$p_1 \times p_2 \times p_3 \times p_4 \Rightarrow \log p_1 + \log p_2 + \log p_3 + \log p_4$$

Generalization and zeros

The Shannon Visualization Method

- Choose a random bigram
  (<s>, w) according to its probability
- Now choose a random bigram
  (w, x) according to its probability
- And so on until we choose </s>
- Then string the words together

```
<s> I
    I want
      want to
           to eat
              eat Chinese
                  Chinese food
                       food  </s>
I want to eat Chinese food
```

Approximating Shakespeare

Unigram

To him swallowed confess hear both. Which. Of save on trail for are ay device and rote life have
Every enter now severally so, let
Hill he late speaks; or! a more to leg less first you enter
Are where exeunt and sighs have rise excellency took of.. Sleep knave we. near; vile like

Bigram

What means, sir. I confess she? then all sorts, he is trim, captain.
Why dost stand forth thy canopy, forsooth; he is this palpable hit the King Henry. Live king. Follow.
What we, hath got so she that I rest and sent to scold and nature bankrupt, nor the first gentleman?

Trigram

Sweet prince, Falstaff shall die. Harry of Monmouth's grave.
This shall forbid it should be branded, if renown made it empty.
Indeed the duke; and had a very good friend.
Fly, and will rid me these news of price. Therefore the sadness of parting, as they say, 'tis done.

Quadrigram

King Henry.What! I will go seek the traitor Gloucester. Exeunt some of the watch. A great banquet serv'd in;
Will you not tell me who I am?
It cannot be but so.
Indeed the short and the long. Marry, 'tis a noble Lepidus.

Shakespeare as corpus

- N=884,647 tokens, V=29,066
- Shakespeare produced 300,000 bigram types out of V^2= 844 million possible bigrams.
  - So 99.96% of the possible bigrams were never seen (have zero entries in the table)
- Quadrigrams worse: What's coming out looks like Shakespeare because it ***is*** Shakespeare

The Wall Street Journal is not Shakespeare

Unigram

Months the my and issue of year foreign new exchange's september were recession exchange new endorsed a acquire to six executives

Bigram

Last December through the way to preserve the Hudson corporation N. B. E. C. Taylor would seem to complete the major central planners one point five percent of U. S. E. has already old M. X. corporation of living on information such as more frequently fishing to keep her

Trigram

They also point to ninety nine point six billion dollars from two hundred four oh six three percent of the rates of interest stores as Mexico and Brazil on market conditions

The perils of overfitting

- N-grams only work well for word prediction if the test corpus looks like the training corpus
  - In real life, it often doesn't
  - We need to train robust models that generalize!
  - One kind of generalization: Zeros!
    - Things that don't ever occur in the training set
      - But occur in the test set

	i	want	to	eat	chinese	food	lunch	spend
i	0.002	0.33	0	0.0036	0	0	0	0.00079
want	0.0022	0	0.66	0.0011	0.0065	0.0065	0.0054	0.0011
to	0.00083	0	0.0017	0.28	0.00083	0	0.0025	0.087
eat	0	0	0.0027	0	0.021	0.0027	0.056	0
chinese	0.0063	0	0	0	0	0.52	0.0063	0
food	0.014	0	0.014	0	0.00092	0.0037	0	0
lunch	0.0059	0	0	0	0	0.0029	0	0
spend	0.0036	0	0.0036	0	0	0	0	0

Zeros

- Training set:
  … denied the allegations
  … denied the reports
  … denied the claims
  … denied the request

  P(“offer” | denied the) = 0

- Test set
  … denied the offer
  … denied the loan

Zero probability bigrams

- Bigrams with zero probability
  - mean that we will assign 0 probability to the test set!
- And hence we cannot compute perplexity (can't divide by 0)!

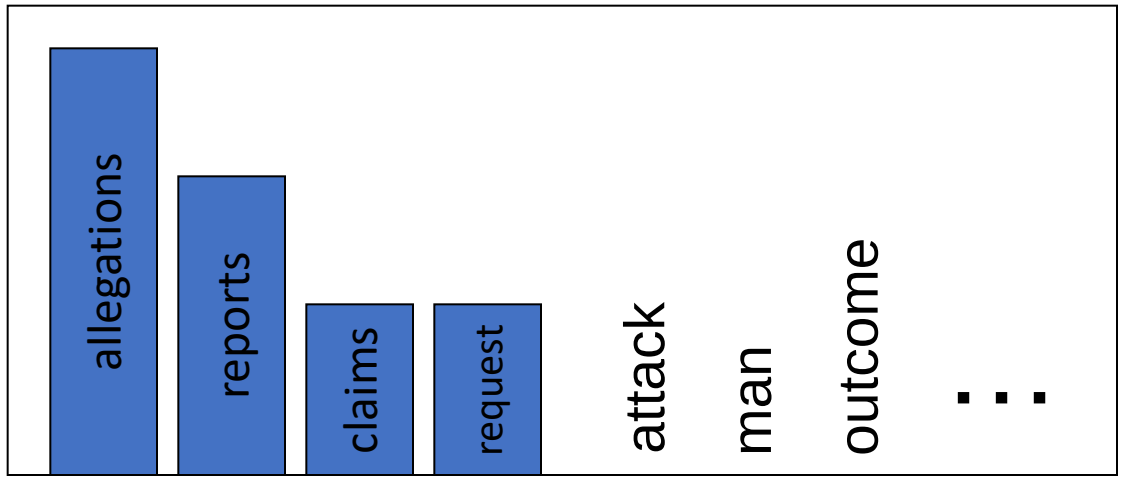
- Zero mitigation
  - Various **smoothing** techniques

Basic Smoothing: Interpolation and Back-off

The intuition of smoothing

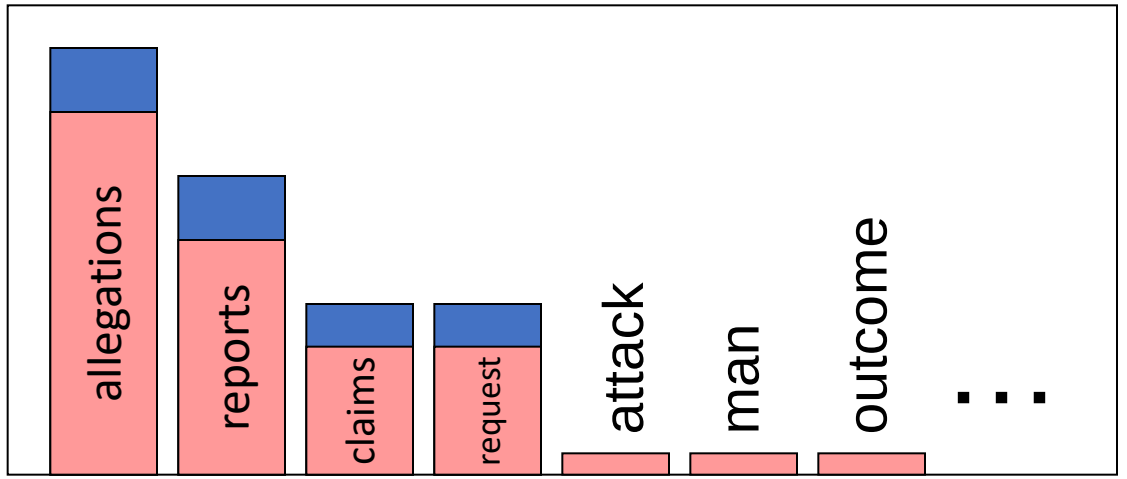
- When we have sparse statistics:

P(w | denied the)
3 allegations
2 reports
1 claims
1 request
7 total



- Steal probability mass to generalize better

P(w | denied the)
2.5 allegations
1.5 reports
0.5 claims
0.5 request
2 other
7 total



Add-one estimation

- Also called Laplace smoothing
- Pretend we saw each word one more time than we did
- Just add one to all the counts!

- MLE estimate:

$$P_{MLE}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

- Add-1 estimate:

$$P_{Add-1}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)+1}{c(w_{i-1})+V}$$

Maximum Likelihood Estimates

- The maximum likelihood estimate
  - of some parameter of a model M from a training set T
  - maximizes the likelihood of the training set T given the model M
- Suppose the word “bagel” occurs 400 times in a corpus of a million words
- What is the probability that a random word from some other text will be “bagel”?
- MLE estimate is 400/1,000,000 = .0004
- This may be a bad estimate for some other corpus
  - But it is the **estimate** that makes it **most likely** that “bagel” will occur 400 times in a million word corpus.

Berkeley Restaurant Corpus: Laplace smoothed bigram counts

	i	want	to	eat	chinese	food	lunch	spend
i	6	828	1	10	1	1	1	3
want	3	1	609	2	7	7	6	2
to	3	1	5	687	3	1	7	212
eat	1	1	3	1	17	3	43	1
chinese	2	1	1	1	1	83	2	1
food	16	1	16	1	2	5	1	1
lunch	3	1	1	1	1	2	1	1
spend	2	1	2	1	1	1	1	1

Laplace-smoothed bigrams

$$P^*(w_n|w_{n-1}) = \frac{C(w_{n-1}w_n) + 1}{C(w_{n-1}) + V}$$

	i	want	to	eat	chinese	food	lunch	spend
i	0.0015	0.21	0.00025	0.0025	0.00025	0.00025	0.00025	0.00075
want	0.0013	0.00042	0.26	0.00084	0.0029	0.0029	0.0025	0.00084
to	0.00078	0.00026	0.0013	0.18	0.00078	0.00026	0.0018	0.055
eat	0.00046	0.00046	0.0014	0.00046	0.0078	0.0014	0.02	0.00046
chinese	0.0012	0.00062	0.00062	0.00062	0.00062	0.052	0.0012	0.00062
food	0.0063	0.00039	0.0063	0.00039	0.00079	0.002	0.00039	0.00039
lunch	0.0017	0.00056	0.00056	0.00056	0.00056	0.0011	0.00056	0.00056
spend	0.0012	0.00058	0.0012	0.00058	0.00058	0.00058	0.00058	0.00058

Reconstituted counts

$$c^*(w_{n-1}w_n) = \frac{[C(w_{n-1}w_n)+1] \times C(w_{n-1})}{C(w_{n-1})+V}$$

	i	want	to	eat	chinese	food	lunch	spend
i	3.8	527	0.64	6.4	0.64	0.64	0.64	1.9
want	1.2	0.39	238	0.78	2.7	2.7	2.3	0.78
to	1.9	0.63	3.1	430	1.9	0.63	4.4	133
eat	0.34	0.34	1	0.34	5.8	1	15	0.34
chinese	0.2	0.098	0.098	0.098	0.098	8.2	0.2	0.098
food	6.9	0.43	6.9	0.43	0.86	2.2	0.43	0.43
lunch	0.57	0.19	0.19	0.19	0.19	0.38	0.19	0.19
spend	0.32	0.16	0.32	0.16	0.16	0.16	0.16	0.16

Compare with raw bigram counts

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

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to	1.9	0.63	3.1	430	1.9	0.63	4.4	133
eat	0.34	0.34	1	0.34	5.8	1	15	0.34
chinese	0.2	0.098	0.098	0.098	0.098	8.2	0.2	0.098
food	6.9	0.43	6.9	0.43	0.86	2.2	0.43	0.43
lunch	0.57	0.19	0.19	0.19	0.19	0.38	0.19	0.19
spend	0.32	0.16	0.32	0.16	0.16	0.16	0.16	0.16

Add-1 estimation is a blunt instrument

- So add-1 isn't used for N-grams:
  - We'll see better methods
- But add-1 is used to smooth other NLP models
  - For text classification
  - In domains where the number of zeros isn't so huge.