Large Language Models

Scaling Laws

ELL881 · AIL821

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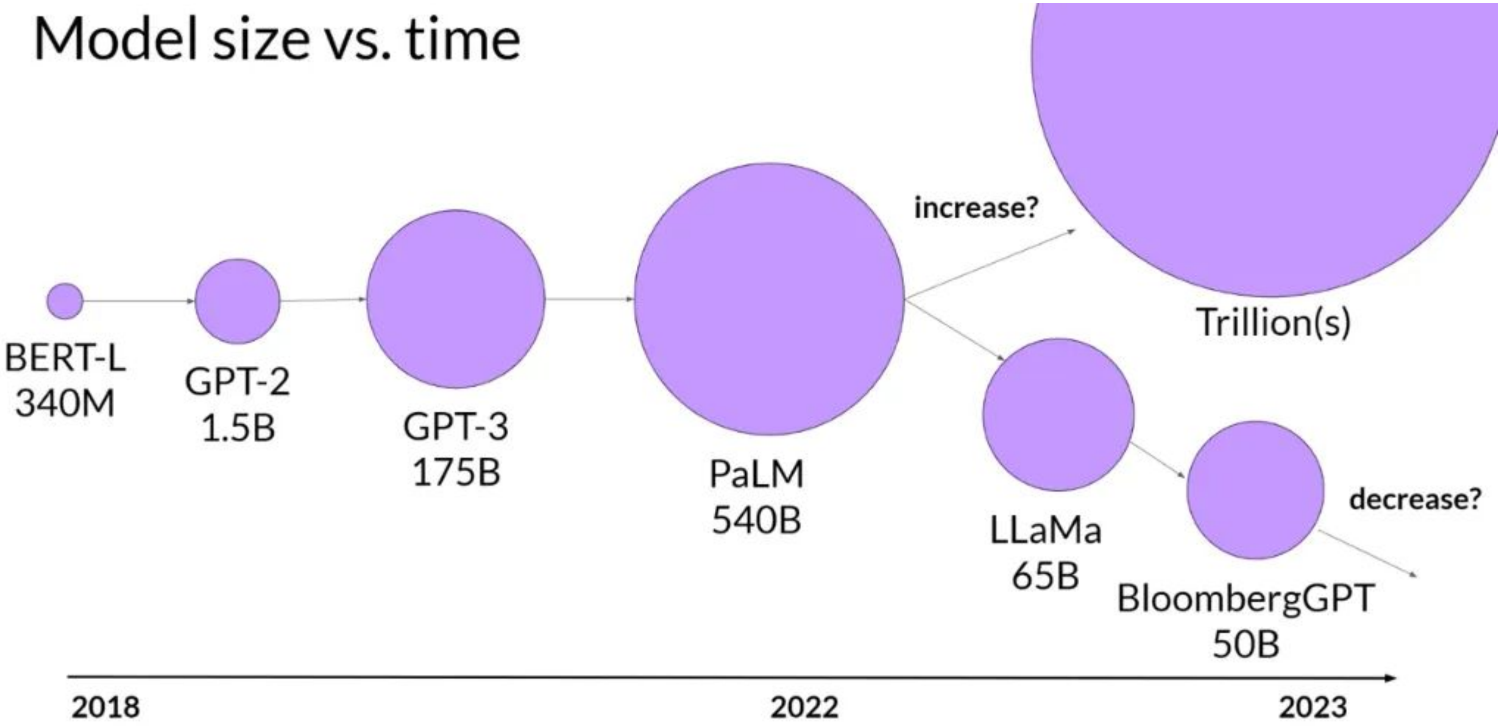
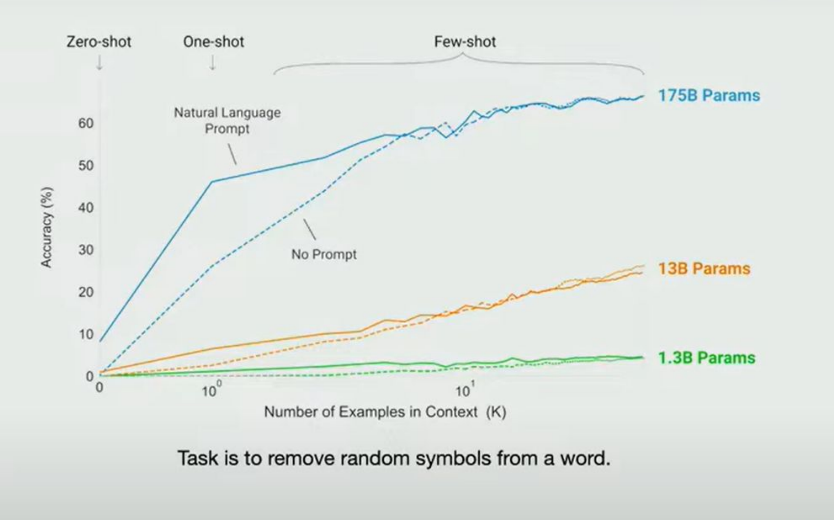
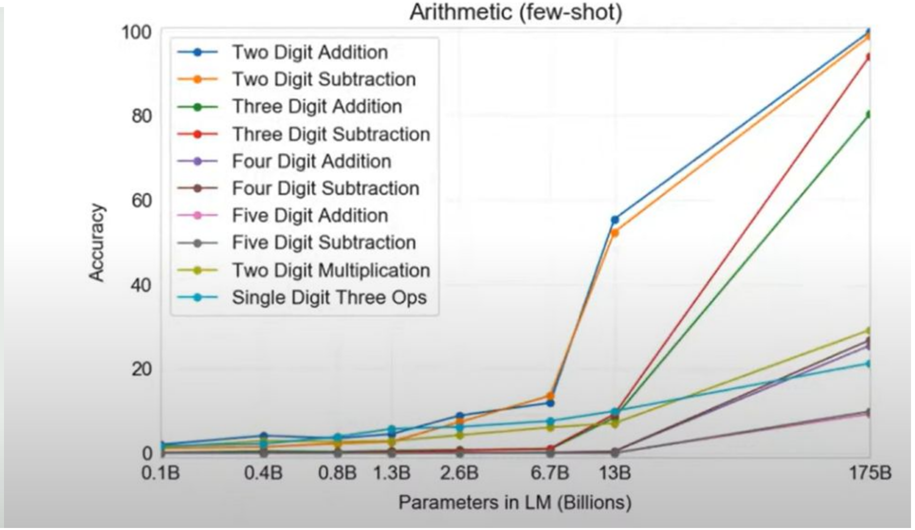
Model size vs. time



“*Emergent*” abilities in LLM

LLMs are few-shot learners (*and larger is better!*)





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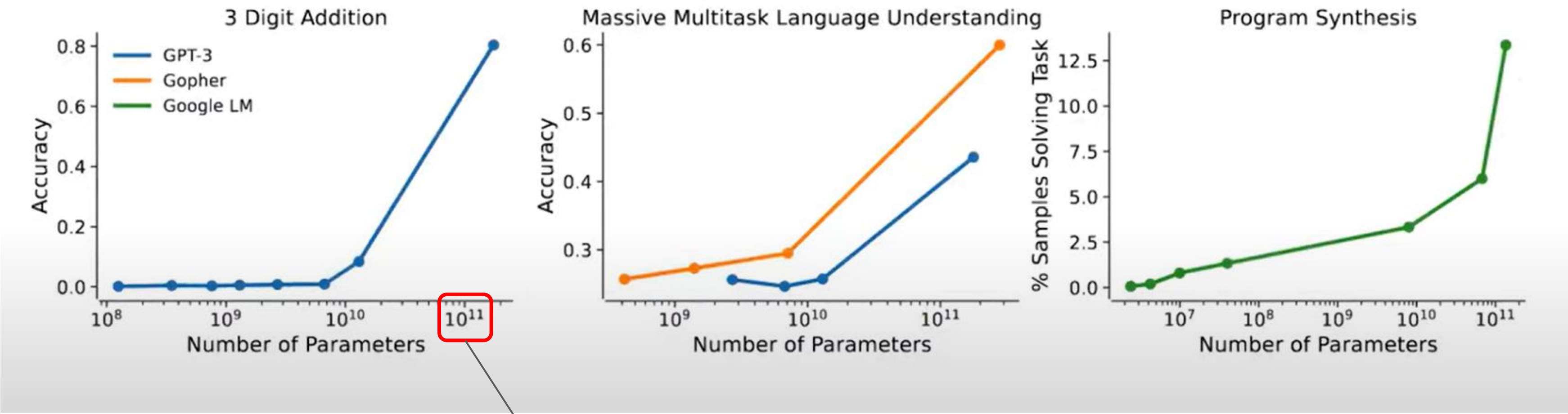
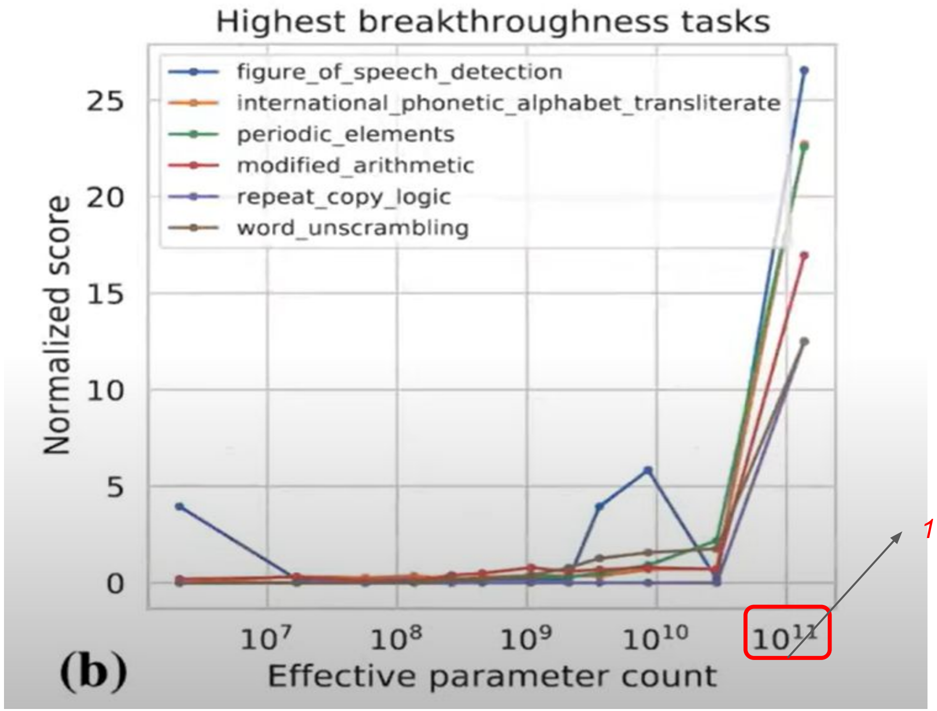


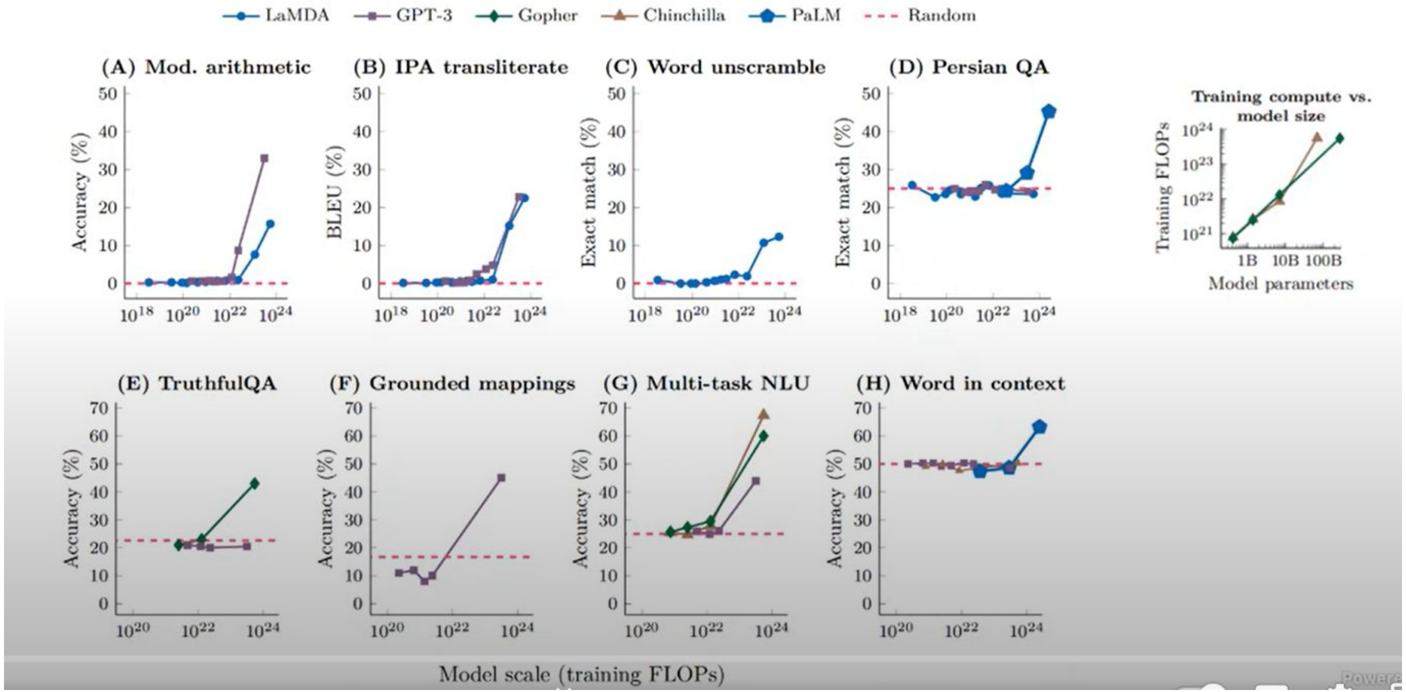
100 billion

LLMs are few-shot learners (*and larger is better!*)



100 billion

LLMs are few-shot learners (*more training is better too!*)



Google BIG-BENCH benchmark

Consider a single Model Family e.g. PaLM

Let x_n be the scale of one family member e.g. PaLM-540B

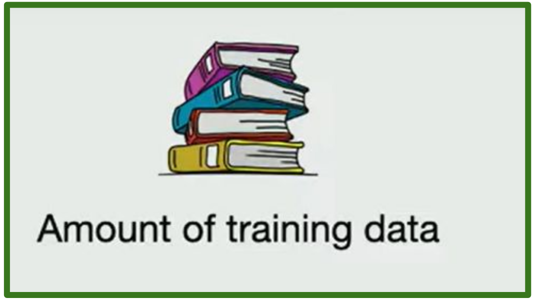
Let y_n be the family member's score on some Task and Metric

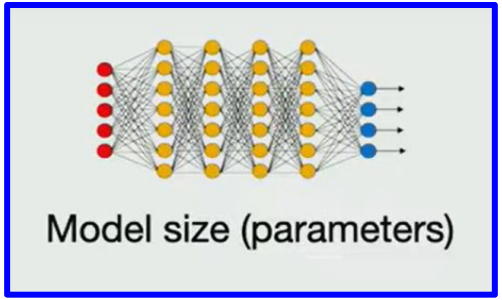
Sort the pairs (x_n, y_n) by model scale x_n, smallest to largest

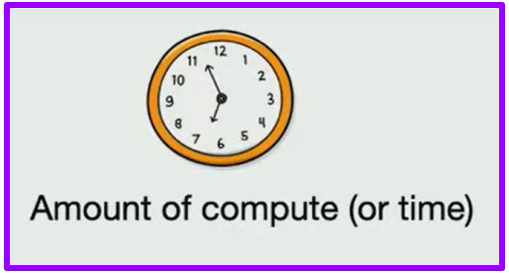
$$\text{Emergence Score}\left(\left\{(x_n, y_n)\right\}_{n=1}^{N}\right) \stackrel{\text{def}}{=} \frac{\text{sign}(\arg\max_i y_i - \arg\min_i y_i)(\max_i y_i - \min_i y_i)}{\sqrt{\text{Median}(\{(y_i - y_{i-1})^2\}_i)}}$$

LLMs "*seems*" to get more intelligent with the following:

NT: scale *w/o bottleneck*







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FLOPs (Floating-point Operations)

Recap on Parameter size & FLOPs

Operation	**Parameters**	**FLOPs per Token**
Embed	$(n_{\text{vocab}} + n_{\text{ctx}})\, d_{\text{model}}$	$4d_{\text{model}}$
Attention: QKV	$n_{\text{layer}} d_{\text{model}} 3 d_{\text{attn}}$	$2 n_{\text{layer}} d_{\text{model}} 3 d_{\text{attn}}$
Attention: Mask	—	$2 n_{\text{layer}} n_{\text{ctx}} d_{\text{attn}}$
Attention: Project	$n_{\text{layer}} d_{\text{attn}} d_{\text{model}}$	$2 n_{\text{layer}} d_{\text{attn}} d_{\text{embd}}$
Feedforward	$n_{\text{layer}} 2 d_{\text{model}} d_{\text{ff}}$	$2 n_{\text{layer}} 2 d_{\text{model}} d_{\text{ff}}$
De-embed	—	$2 d_{\text{model}} n_{\text{vocab}}$
Total (Non-Embedding)	$N = 2 d_{\text{model}} n_{\text{layer}} (2 d_{\text{attn}} + d_{\text{ff}})$	$C_{\text{forward}} = 2N + 2 n_{\text{layer}} n_{\text{ctx}} d_{\text{attn}}$

one PF-day $= 10^{15} \times 24 \times 3600 = 8.64 \times 10^{19}$ floating point operations.

Emergent abilities are *unpredictable*

If that's true, we never get to know the following:

- *<u>Which</u>* abilities (and *<u>when</u>*) exactly will emerge?
- *<u>What</u>* controls the trigger?
- Can we make <u>desirable abilities</u> to emerge *faster*?
- Can we make <u>undesirable abilities</u> to be <u>suppressed</u>?

Is the value of scaling laws only in predicting?

- How much *return* for a given compute (resource) budget?
- How to *allocate* the compute budget - model size vs. dataset size?

Can there be a curve that fits "emergence"? *Intuition*

Input: $x_1 \ \ldots x_n \ \sim N(\mu, \sigma^2)$

Task: estimate the average as $\hat{\mu} = \frac{\sum_i x_i}{n}$

What's the error? By standard arguments..

$$\mathrm{E}[(\hat{\mu} - \mu)^2] = \frac{\sigma^2}{n}$$

This is a scaling law!!

$$\log(Error) = -\log n + 2 \log \sigma$$

More generally, any polynomial rate $1/n^{\alpha}$ is a scaling law

Source: CS-324, Stanford University

What about fitting "emergence" in *non-parametric* setting

Neural nets can approximate arbitrary functions. Lets turn that into an example.

Input: $x_1 \dots x_n$ uniform in 2D unit box. $y_i = f(x_i) + N(0,1)$
Task: estimate f(x)
Approach: cut up the 2D space into boxes with length $n^{-\frac{1}{4}}$, average in each box

What's our estimation error?

Informally, we have $\sqrt{n}$ boxes, each box gets $\sqrt{n}$ samples.

$$Error \approx \frac{1}{\sqrt{n}} + (other\ smoothness\ terms)$$

In d-dimensions, this becomes $Error = n^{-1/d}$ - **This means scaling is** $\boldsymbol{y = -\frac{1}{d}x + C}$

Takeaway: flexible 'nonparametric' learning has dimension dependent scaling laws.

Source: CS-324, Stanford University

Notations

$C \approx 6N$ floating point operators per training token.

$C = 6NBS$ estimates the (non-embedding) compute used at batch size B

S is the number of training steps (ie parameter updates)

$S_{\min}$ – an estimate of the minimal number of training steps needed to reach a given value of the loss. This is also the number of training steps that would be used if the model were trained at a batch size much greater than the critical batch size.

one PF-day $= 10^{15} \times 24 \times 3600 = 8.64 \times 10^{19}$ floating point operations.

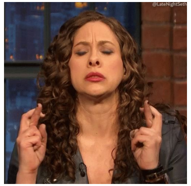
B_{crit} – the critical batch size [MKAT18], defined and discussed in Section 5.1. Training at the critical batch size provides a roughly optimal compromise between time and compute efficiency.

$C_{\min}$ – an estimate of the minimum amount of non-embedding compute to reach a given value of the loss. This is the training compute that would be used if the model were trained at a batch size much less than the critical batch size.

2020 Arxiv (cs);

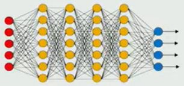
What if the loss-drop (i.e., emergence) follows power-law?

$y = ax^k$ → Power Law: *can be -ve* → $L(X) \propto 1/X^{\alpha_X}$



$$L(N) \approx \left(\frac{N_c}{N}\right)^{\alpha_N}$$

$\alpha_N \sim 0.076, \quad N_c \sim 8.8 \times 10^{13}$ (non-embedding parameters)



Model size (parameters)

$$L(D) \approx \left(\frac{D_c}{D}\right)^{\alpha_D}$$

$\alpha_D \sim 0.095, \quad D_c \sim 5.4 \times 10^{13}$ (tokens)

Amount of training data

Kaplan Laws

$$L(C) \approx \left(\frac{C_c}{C}\right)^{\alpha_C}$$

$\alpha_C = 0.057 \quad C_c = 1.6 \times 10^7$ PF-days

Amount of compute (or time)

$$L(C_{\min}) = \left(C_c^{\min}/C_{\min}\right)^{\alpha_C^{\min}}$$

$\alpha_C^{\min} \sim 0.050, \quad C_c^{\min} \sim 3.1 \times 10^8$ (PF-days)

A more reliable version:

$$L(C) \approx \left(\frac{C_c}{C}\right)^{\alpha_C}$$

$$L(C_{\min}) = \left(C_c^{\min}/C_{\min}\right)^{\alpha_C^{\min}}$$

Parameters	Data	Compute	Batch Size
Optimal	∞	C	Fixed

$$\alpha_C^{\min} \sim 0.050, \quad C_c^{\min} \sim 3.1 \times 10^8 \text{ (PF-days)}$$

vs.

N_{opt}	D_{opt}	$C_{\min}$	$B \ll B_{\text{crit}}$

$$C_{\min}(C) \equiv \frac{C}{1 + B/B_{\text{crit}}(L)} \qquad \text{(minimum compute, at } B \ll B_{\text{crit}})$$

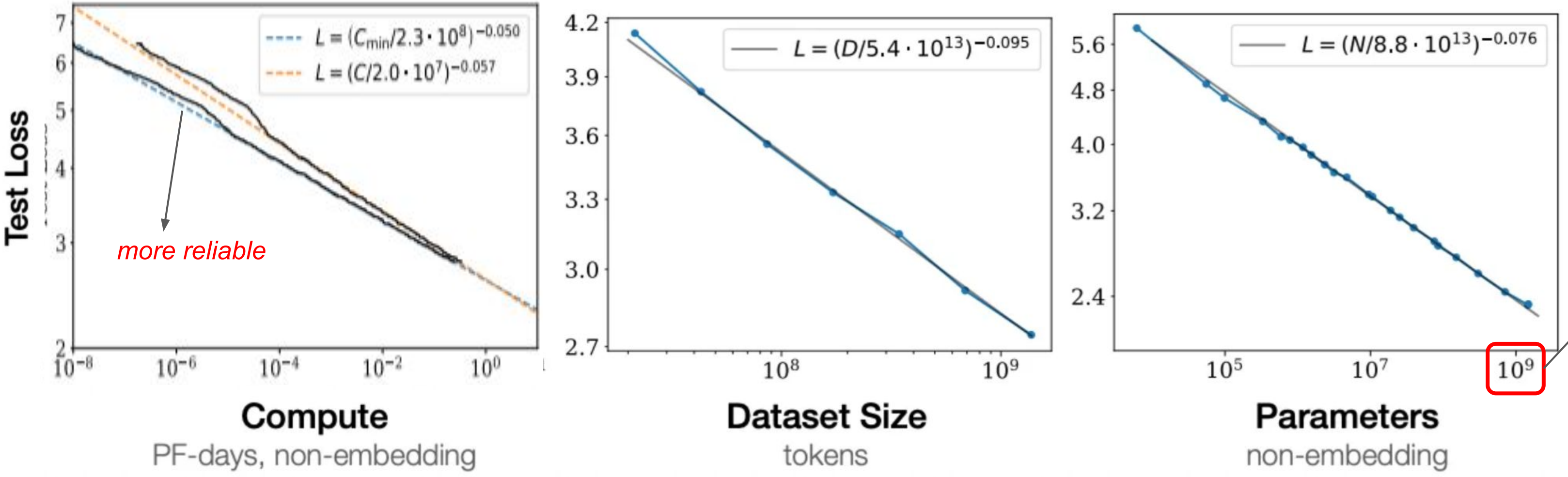
$$B_{\text{crit}}(L) \approx \frac{B_*}{L^{1/\alpha_B}}$$

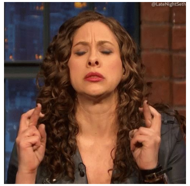
$$B_* \sim 2 \cdot 10^8 \text{ tokens}, \quad \alpha_B \sim 0.21$$

Compute-Efficient Value	Power Law	Scale
$N_{\text{opt}} = N_e \cdot C_{\min}^{p_N}$	$p_N = 0.73$	$N_e = 1.3 \cdot 10^9$ params
$B \ll B_{\text{crit}} = \frac{B_*}{L^{1/\alpha_B}} = B_e C_{\min}^{p_B}$	$p_B = 0.24$	$B_e = 2.0 \cdot 10^6$ tokens

Kaplan Laws

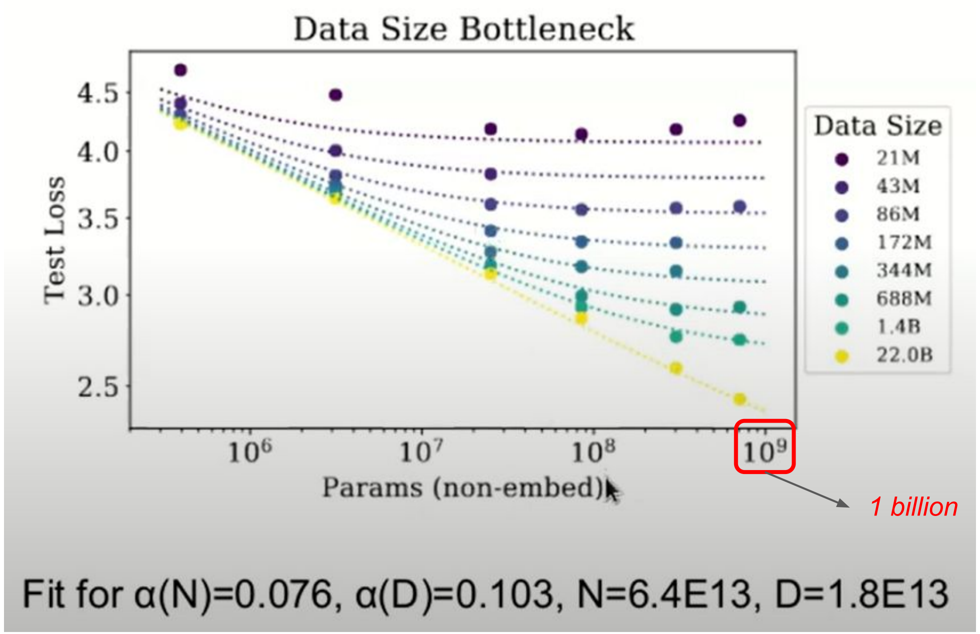
We are in luck! Turns out that scale ***is*** predictable



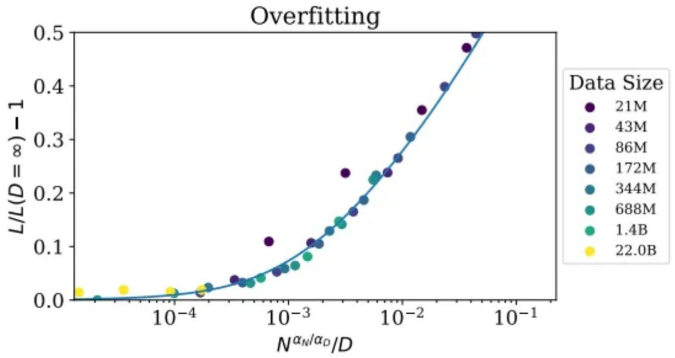


(NT: log-log plotting) → Now that's smooth!!

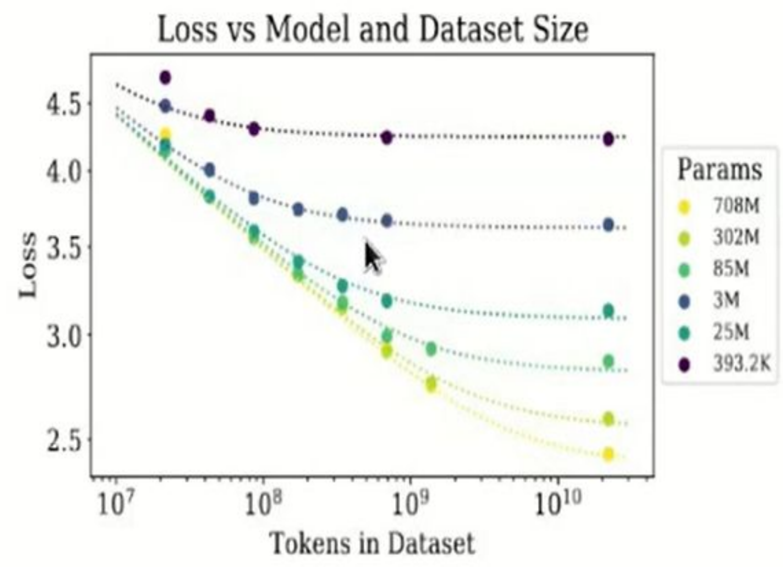
Observation 1a: *Universality of Overfitting*

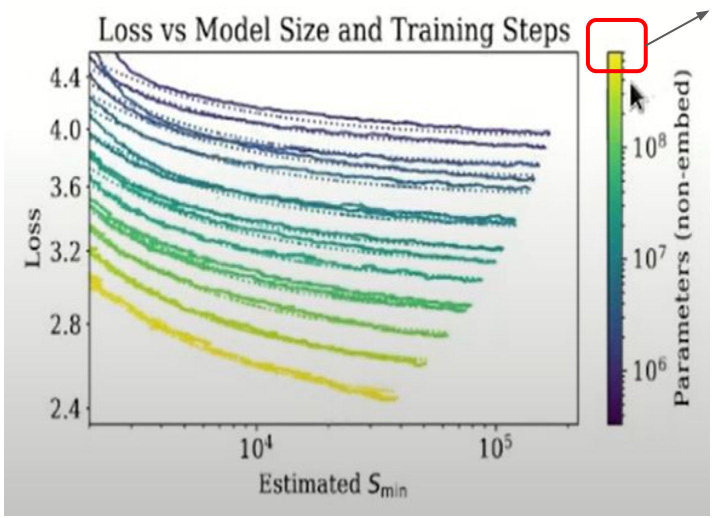


1 billion



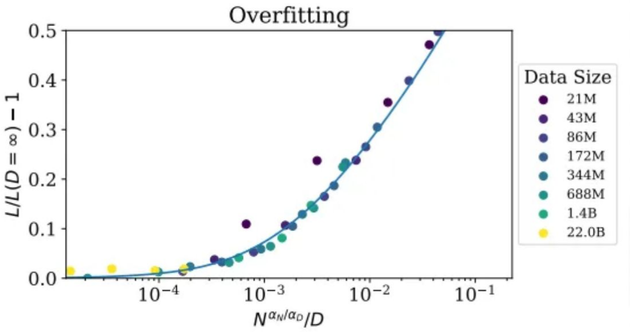
Observation 1b: *Sample Efficiency*





1 billion

Key takeaway 1: <u>*Both*</u> parameter and dataset to be scaled



$$D \propto N^{\frac{\alpha_N}{\alpha_D}} \sim N^{0.74}$$

Performance penalty is $N^{0.75}$ / D

- if model increases 8x, dataset must increase 5x

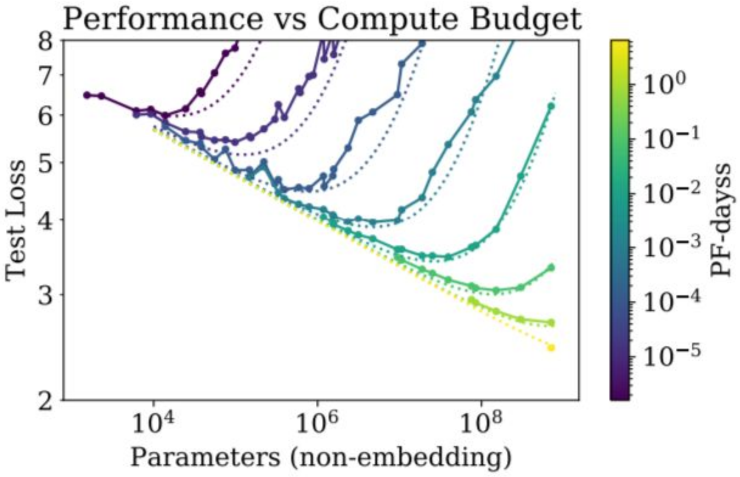
Both needs to be scaled <u>together</u>

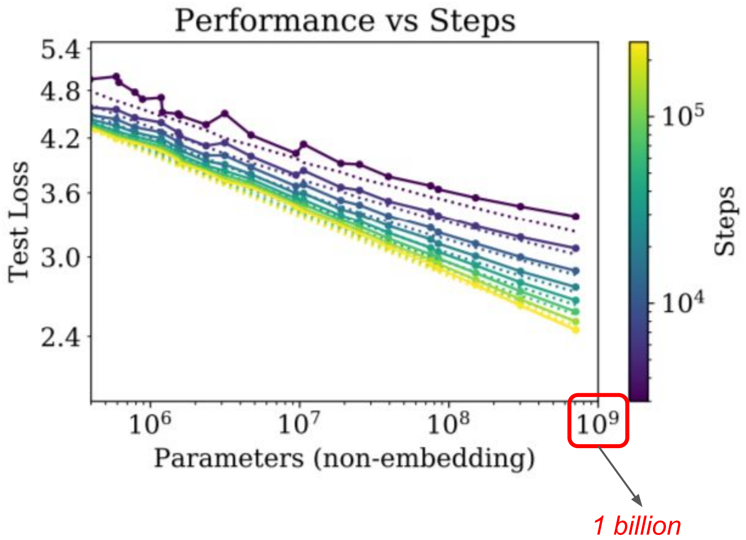
$$L(N) \approx \left(\frac{N_c}{N}\right)^{\alpha_N}$$

$$L(D) \approx \left(\frac{D_c}{D}\right)^{\alpha_D}$$

$$L(N, D) = \left[\left(\frac{N_c}{N}\right)^{\frac{\alpha_N}{\alpha_D}} + \frac{D_c}{D}\right]^{\alpha_D}$$

Observation 2: What about training time (*steps & FLOPs*)?





Key takeaway 2: *Universality of training*

$$L(N,S) = \left(\frac{N_c}{N}\right)^{\alpha_N} + \left(\frac{S_c}{S_{\min}(S)}\right)^{\alpha_S}$$

$$L(N,D) = \left[\left(\frac{N_c}{N}\right)^{\frac{\alpha_N}{\alpha_D}} + \frac{D_c}{D}\right]^{\alpha_D}$$

$S_c \approx 2.1 \times 10^3$ and $\alpha_S \approx 0.76$, and $S_{\min}(S)$ is the minimum possible number of optimization steps

$$S_{\min}(S) \equiv \frac{S}{1 + B_{\mathrm{crit}}(L)/B} \qquad \text{(minimum steps, at } B \gg B_{\mathrm{crit}}\text{)}$$

$$B_{\mathrm{crit}}(L) \approx \frac{B_*}{L^{1/\alpha_B}}$$

$$B_* \sim 2 \cdot 10^8 \text{ tokens}, \quad \alpha_B \sim 0.21$$

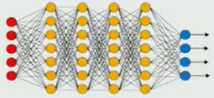
Parameter	α_N	α_D	N_c	D_c
Value	0.076	0.103	6.4×10^{13}	1.8×10^{13}

Parameter	α_N	α_S	N_c	S_c
Value	0.077	0.76	6.5×10^{13}	2.1×10^3

Kaplan Laws

Are we only to worry about

Amount of training data

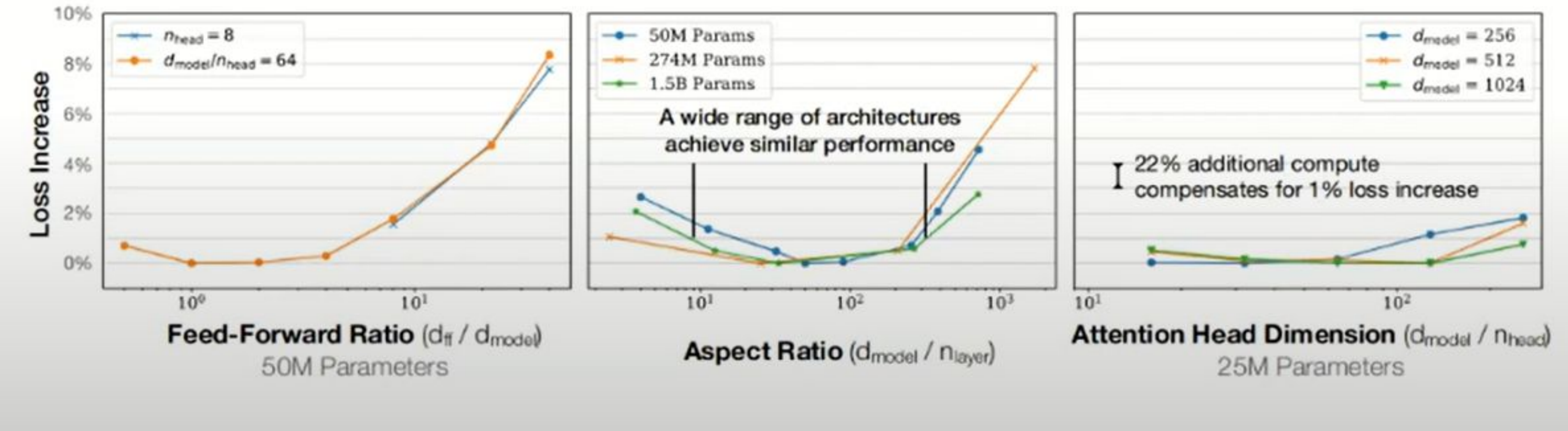


Model size (parameters)

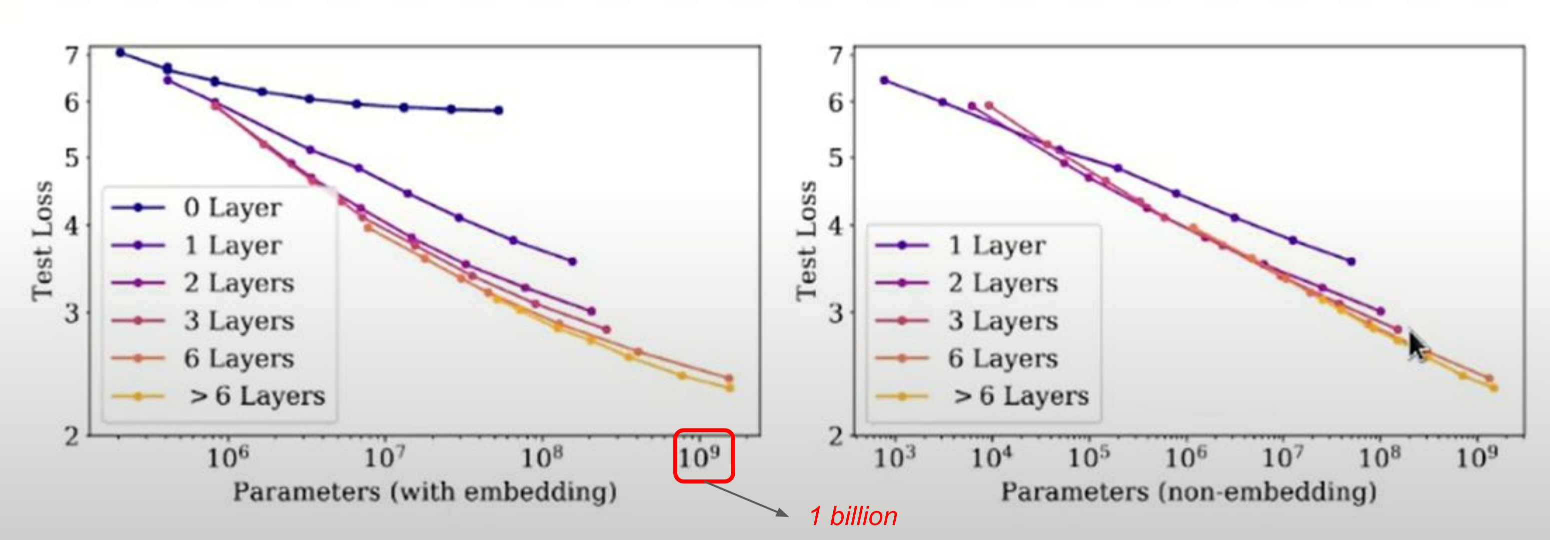
Amount of compute (or time)

???

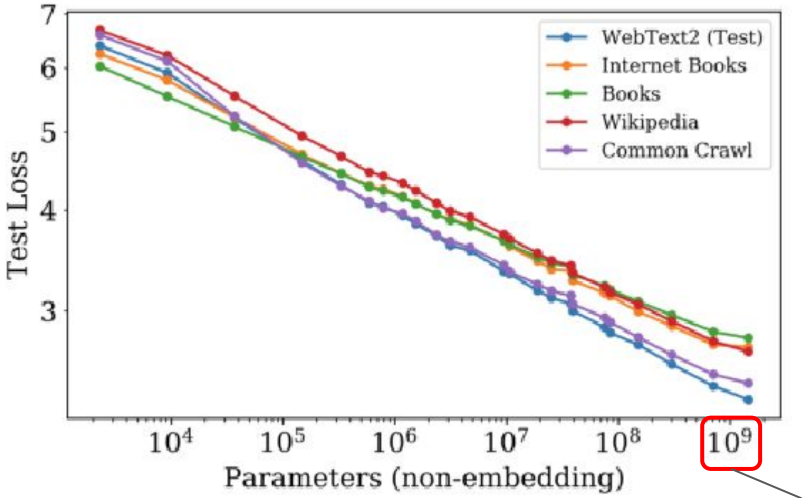
Key Takeaway 3: Model shape does not matter!



Key Takeaway 4: Embedding matrix does not matter!



Key Takeaway 5: Dataset *composition* does not matter!



1 billion

Kaplan Scaling Laws at a glance:

Power Law	Scale (tokenization-dependent)
$\alpha_N = 0.076$	$N_c = 8.8 \times 10^{13}$ params (non-embed)
$\alpha_D = 0.095$	$D_c = 5.4 \times 10^{13}$ tokens
$\alpha_C = 0.057$	$C_c = 1.6 \times 10^{7}$ PF-days
$\alpha_C^{\min} = 0.050$	$C_c^{\min} = 3.1 \times 10^{8}$ PF-days
$\alpha_B = 0.21$	$B_* = 2.1 \times 10^{8}$ tokens
$\alpha_S = 0.76$	$S_c = 2.1 \times 10^{3}$ steps

Parameters	Data	Compute	Batch Size	Equation
N	∞	∞	Fixed	$L(N) = (N_c/N)^{\alpha_N}$
∞	D	Early Stop	Fixed	$L(D) = (D_c/D)^{\alpha_D}$
Optimal	∞	C	Fixed	$L(C) = (C_c/C)^{\alpha_C}$ (naive)
N_{opt}	D_{opt}	$C_{\min}$	$B \ll B_{\text{crit}}$	$L(C_{\min}) = \left(C_c^{\min}/C_{\min}\right)^{\alpha_C^{\min}}$
N	D	Early Stop	Fixed	$L(N,D) = \left[\left(\frac{N_c}{N}\right)^{\frac{\alpha_N}{\alpha_D}} + \frac{D_c}{D}\right]^{\alpha_D}$
N	∞	S steps	B	$L(N,S) = \left(\frac{N_c}{N}\right)^{\alpha_N} + \left(\frac{S_c}{S_{\min}(S,B)}\right)^{\alpha_S}$

Is there any other alternative law?

Turns out there is!

Lower bound

$$\text{Loss}(N_T, D) = \frac{N_c}{N_T^{\alpha}} + \frac{D_c}{D^{\beta}} + E,$$

$$\text{Loss}(N_T, C_T) = \frac{N_c}{N_T^{\alpha}} + \frac{D_c}{(C_T/6N_T)^{\beta}} + E$$

<u>Chinchilla (Hoffman) Scaling Law</u>

The Chinchilla (Hoffman) Scaling Law

$$\mathrm{Loss}(N_T, D) = \frac{N_c}{N_T^{\alpha}} + \frac{D_c}{D^{\beta}} + E$$

$$L(N, D) = 1.69 + \frac{406.4}{N^{0.34}} + \frac{410.7}{D^{0.28}}$$

$$N_{opt}(C) = G(C/6)^{a} \quad D_{opt}(C) = G^{-1}(C/6)^{b}$$

$$\text{where} \quad G = \left(\frac{\alpha A}{\beta B}\right)^{\frac{1}{\alpha+\beta}} \quad a = \frac{\beta}{\alpha+\beta} \quad b = \frac{\alpha}{\alpha+\beta}$$

Fitting the constants, yields: $\alpha \approx \beta$

i.e. equal scaling of **N** and **D**.

Chinchilla Scaling Law vs. Kaplan Scaling Law

$$\text{Kaplan:} N^*_{\backslash E} \propto C^{0.73}_{\backslash E}$$

$$\text{Chinchilla:} N^*_T \propto C^{0.50}_T.$$

Performance penalty is $N^{0.75}$ / D

- if model increases 8x, dataset must increase 5x

vs.

Fitting the constants, yields: $\alpha \approx \beta$

i.e. equal scaling of ***N*** and ***D***.

$$N_T = N_E + N_{\backslash E}, \qquad C_T = 6N_T D = 6(N_E + N_{\backslash E})D,$$

$$N_E = (h + v)d, \qquad C_{\backslash E} = 6N_{\backslash E}D.$$

The (revised) Chinchilla Scaling Law

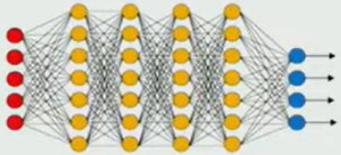
$$L(N,D) = 1.82 + \frac{514.0}{N^{0.35}} + \frac{2115.2}{D^{0.37}}$$

$$L(N,D) = 1.69 + \frac{406.4}{N^{0.34}} + \frac{410.7}{D^{0.28}}$$

Is it a problem with our point-of-*view*?

LLMs "*seems*" to get more intelligent with the following:

Amount of training data

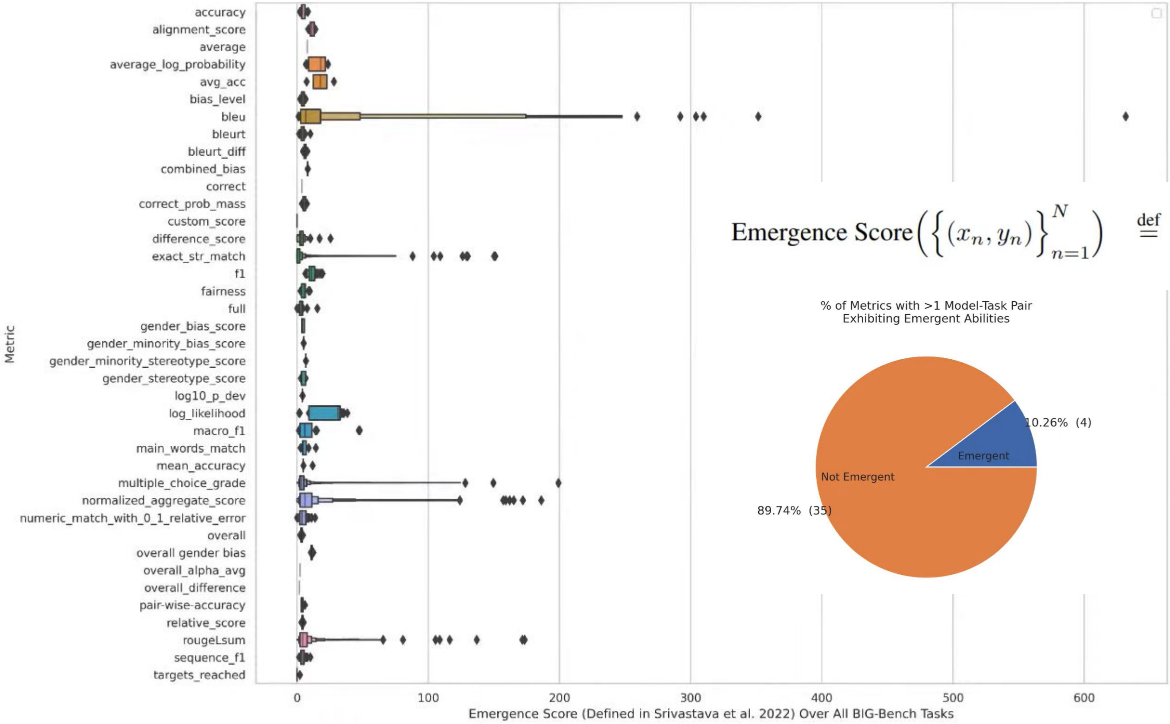


Model size (parameters)

$$\mathcal{L}_{CE}(N) = \left(\frac{N}{c}\right)^{\alpha}$$

Amount of compute (or time)

Motivation: Not all metrics score same (Emergence Score)



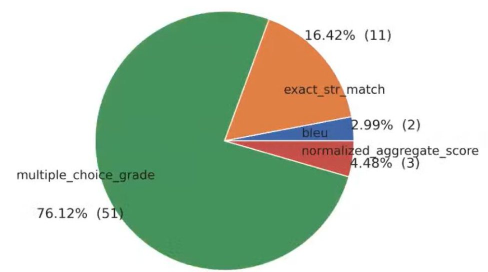
$$\text{Emergence Score}\left(\left\{(x_n, y_n)\right\}_{n=1}^{N}\right) \stackrel{\text{def}}{=} \frac{\text{sign}(\arg\max_i y_i - \arg\min_i y_i)(\max_i y_i - \min_i y_i)}{\sqrt{\text{Median}(\{(y_i - y_{i-1})^2\}_i)}}$$

Is your accuracy metric non-linear or discontinuous?

> 92% of BIG-BENCH:

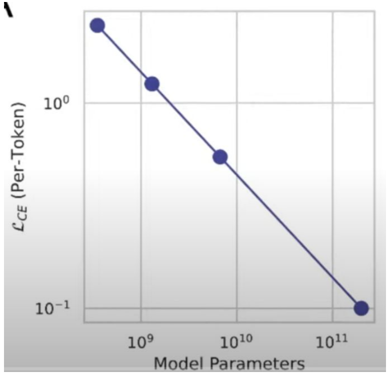
$$\text{Multiple Choice Grade} \stackrel{\text{def}}{=} \begin{cases} 1 & \text{if highest probability mass on correct option} \\ 0 & \text{otherwise} \end{cases}$$

$$\text{Exact String Match} \stackrel{\text{def}}{=} \begin{cases} 1 & \text{if output string exactly matches target string} \\ 0 & \text{otherwise} \end{cases}$$



Too challenging for smaller models!
Is it really worth??

Power Law in play!

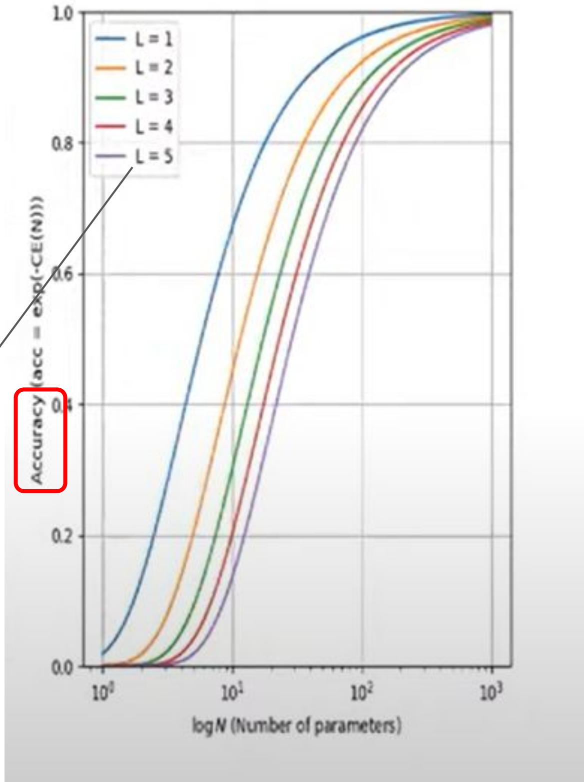


$$\mathcal{L}_{CE}(N) = \left(\frac{N}{c}\right)^{\alpha} = -\log \hat{p}_{v^*}(N)$$

$$\hat{p}_{v^*}(N) = \exp\left(-(N/c)^{\alpha}\right)$$

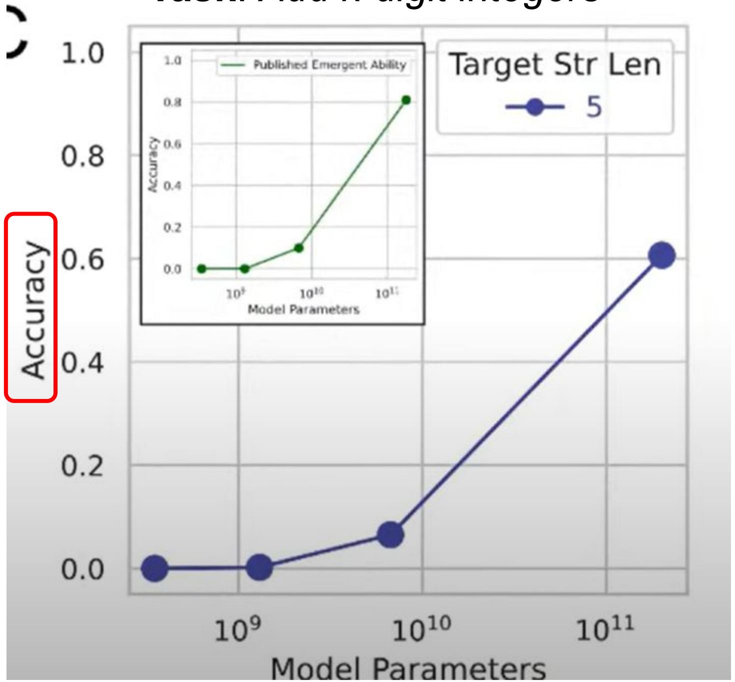
$$L(D_0, N) = B_0 + \frac{B_1}{N^{\alpha_W}} + \ldots$$

Length of token



Problem with Non-linear Measure: Eg.: *Exact string match*

Task: *Add k-digit integers*



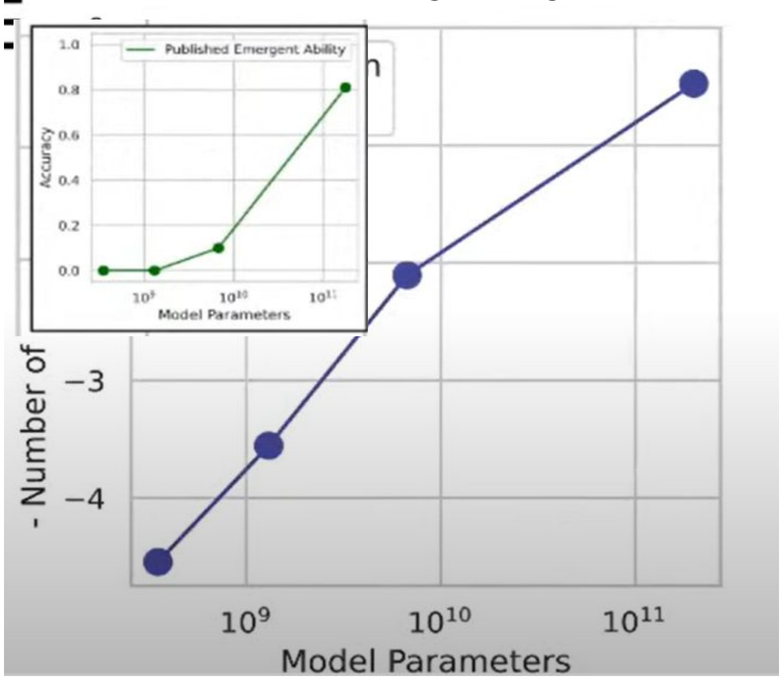
1	if all K+1 digits in model's output are correct
0	otherwise

$$\hat{p}_{v^*}(N) = \exp\left(-(N/c)^{\alpha}\right)$$

$$\text{Accuracy}(N) \approx p_N(\text{single token correct})^{\text{num. of tokens}}$$

Change of perspective: Measure: *Edit distance*

Task: *Add k-digit integers*



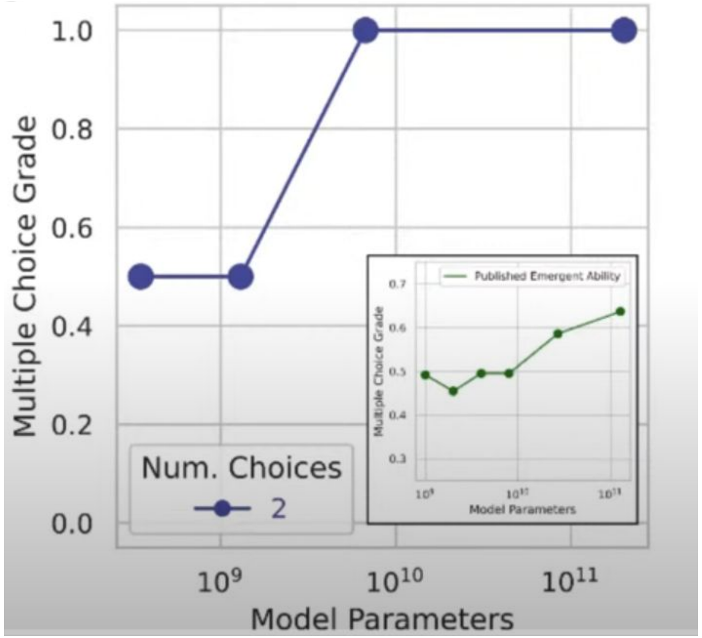
1	if all K+1 digits in model's output are correct
0	otherwise

$$\hat{p}_{v^*}(N) = \exp\Big(-(N/c)^{\alpha} \Big)$$

$$\text{Edit Distance}(N) \approx L\Big(1 - p_N(\text{single token correct})\Big)$$

Problem with Discontinuous Measure: Eg.: *MCG*

Task: *Choose one of two*

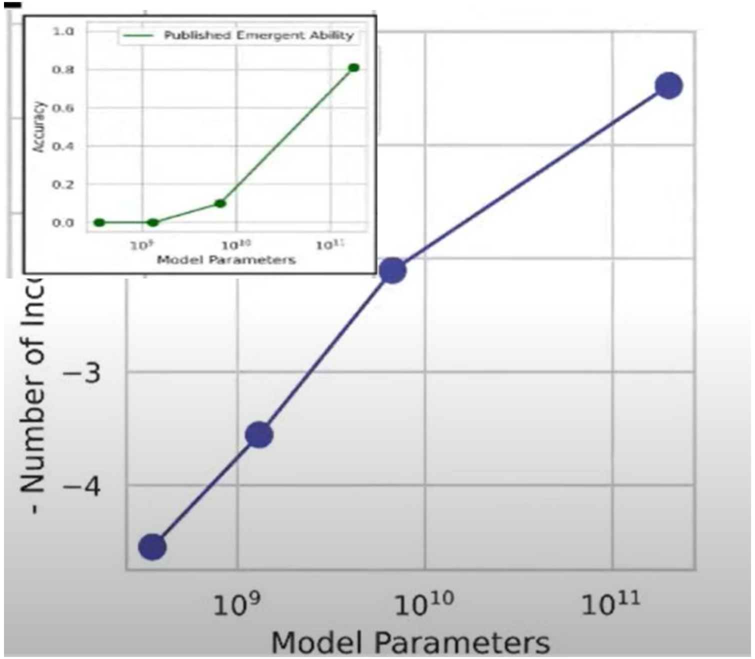


1	if highest probability mass on correct option
0	otherwise

$$\hat{p}_{v^*}(N) = \exp\left(-(N/c)^{\alpha}\right)$$

Change of perspective: Measure: *Brier Score*

Task: *Choose one of two*

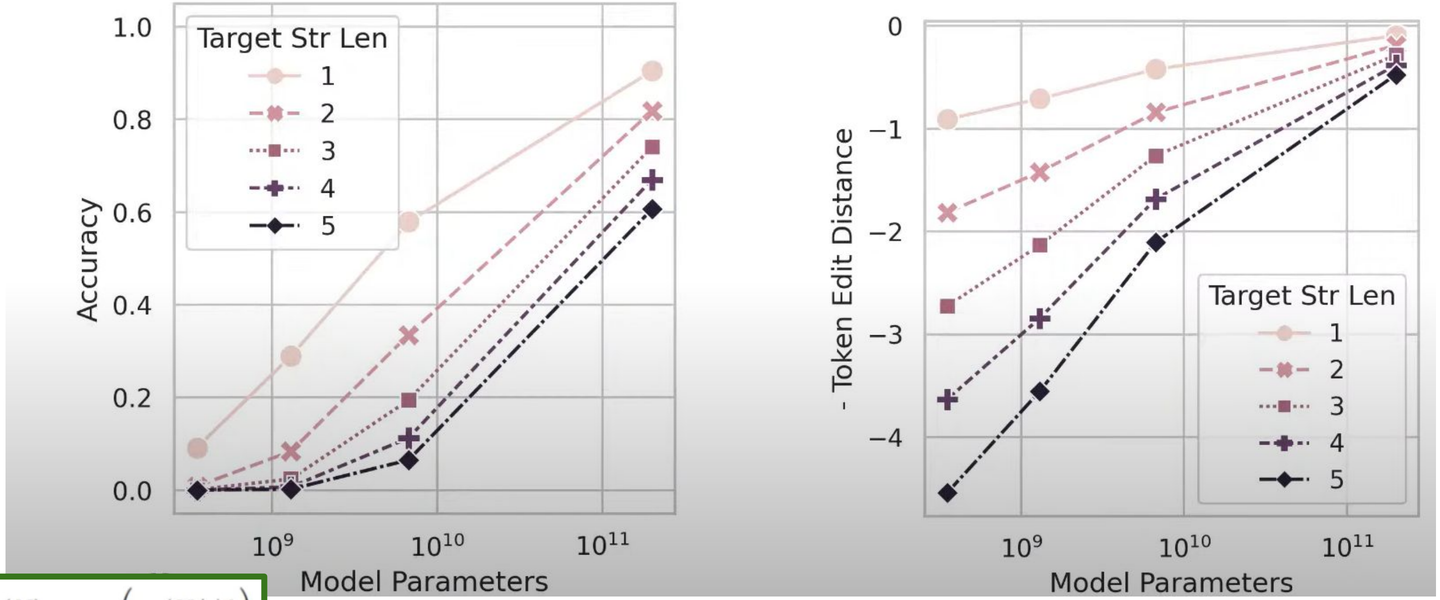


1	if all K+1 digits in model's output are correct
0	otherwise

$$\hat{p}_{v^*}(N) = \exp\left(-(N/c)^\alpha\right)$$

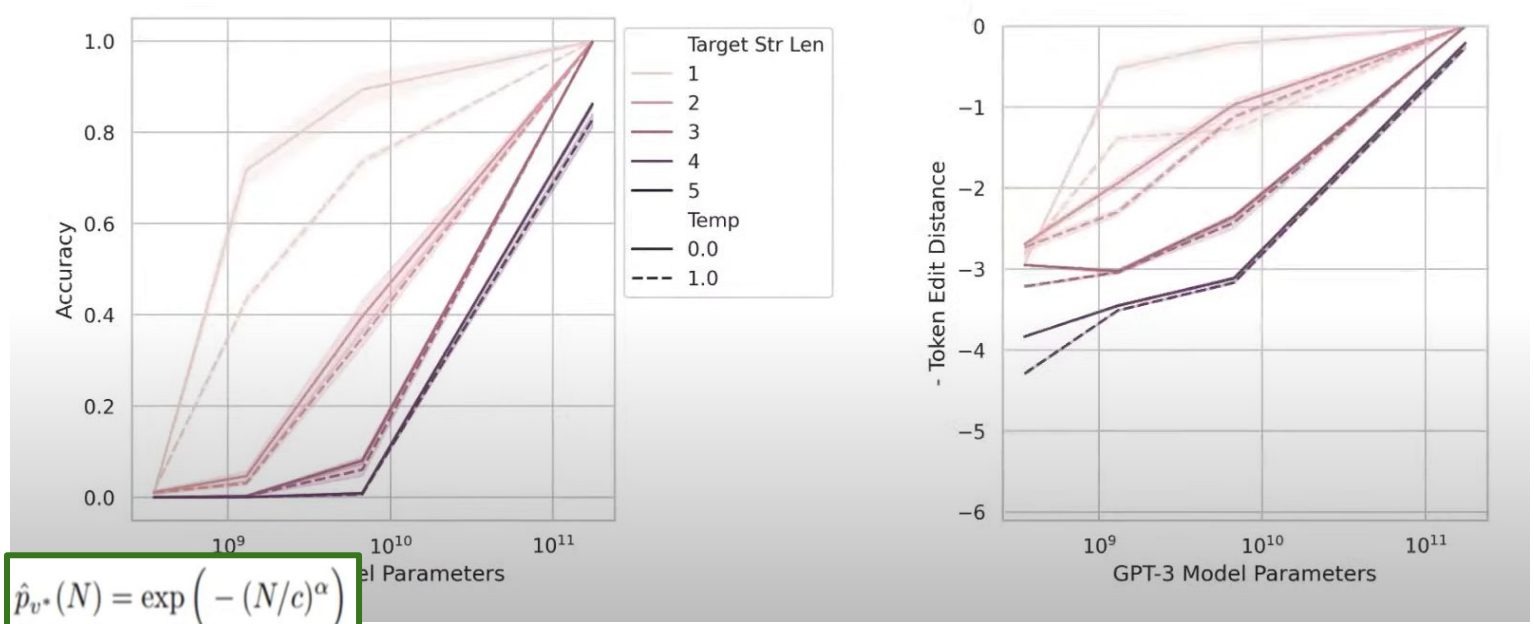
Brier Score = (1 – probability mass on correct option)^2

Prediction: Power Law vs. Near-Linear counterpart



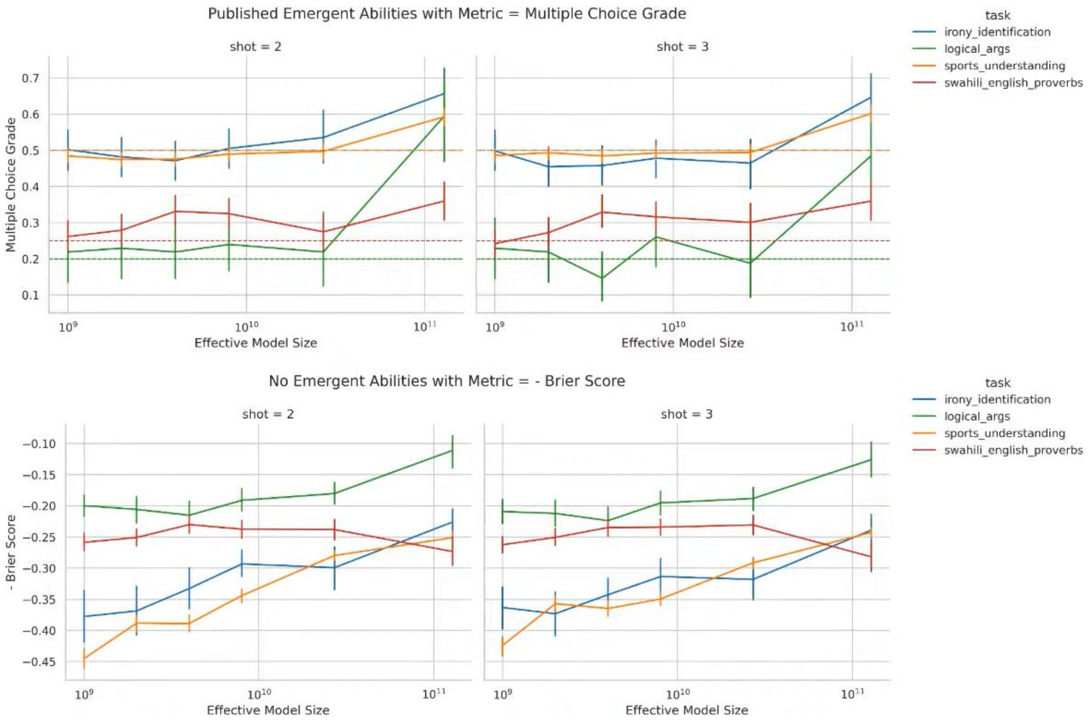
$$\hat{p}_{v^*}(N) = \exp\left(-(N/c)^\alpha\right)$$

Results on GPT3.5/3: Task: 2-digit integer multiplication



$$\hat{p}_{v^*}(N) = \exp\left(-(N/c)^{\alpha}\right)$$

Does the claim work for Google BIG-BENCH benchmark?



Key Takeaways

- Want to predict without the theatrics? Choose a *metric that's "soft"* (in the continuous sense)
- There's *no sudden jump* in reality ("*most*" can be predicted on a near-linear scale)
- Do we really need the power law of scale? Maybe not!