Introduction to Mixture of Experts (MoEs)

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Large Language Models: Introduction and Recent Advances

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IBM Research, India
Conversational-AI

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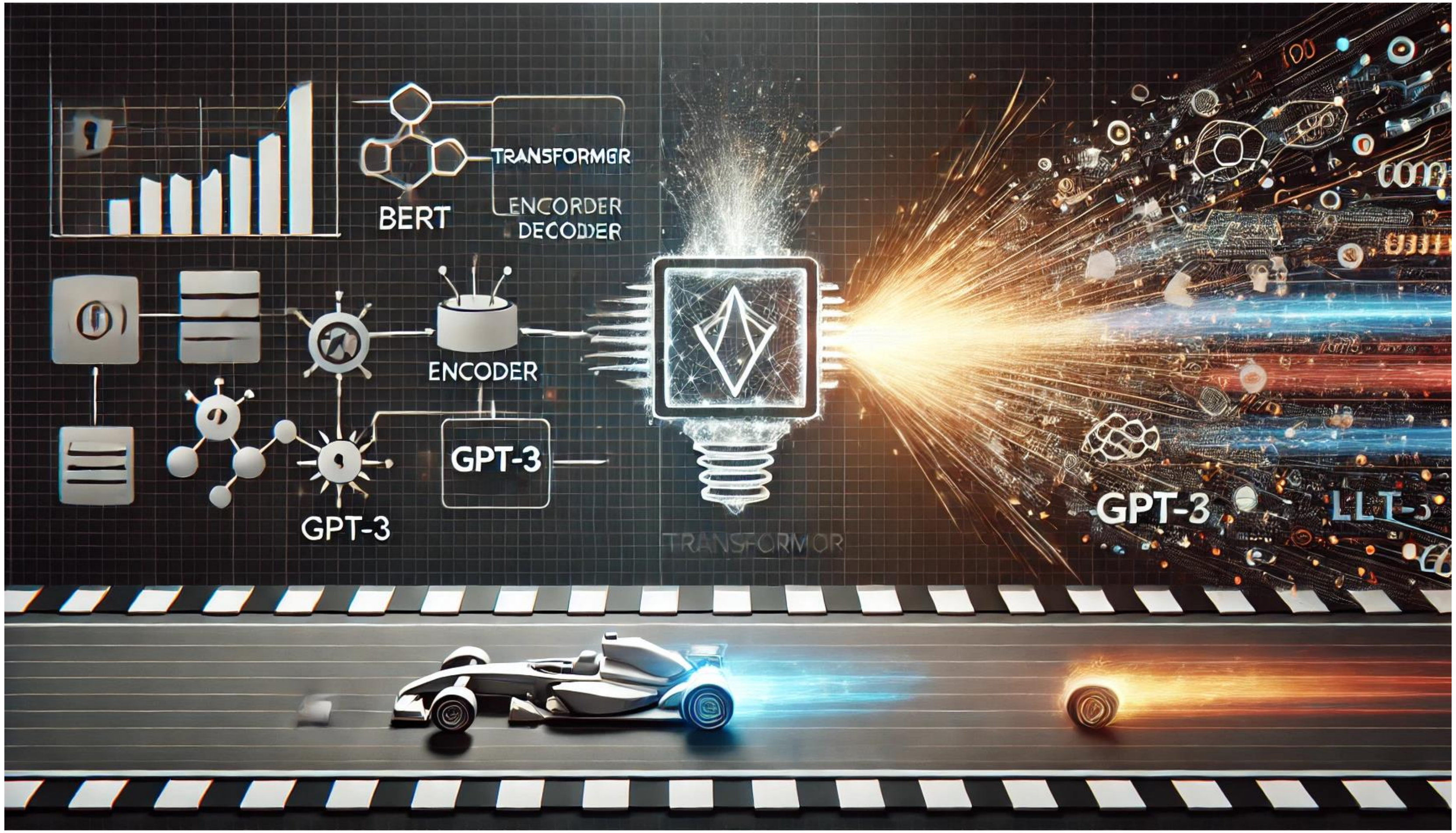
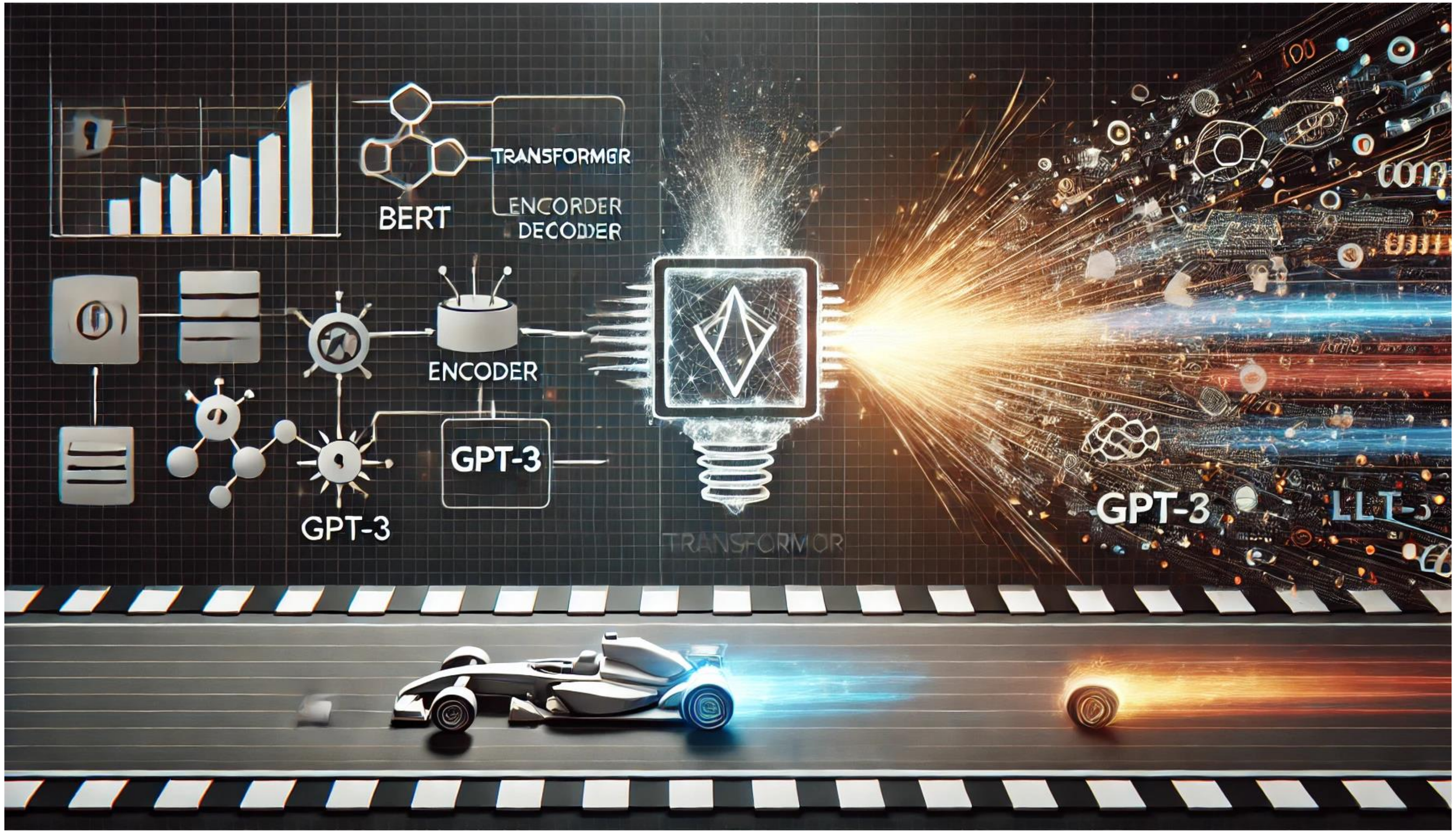
Sachindra Joshi

Large Language Models



Content credits: Chat GPT

Large Language Models

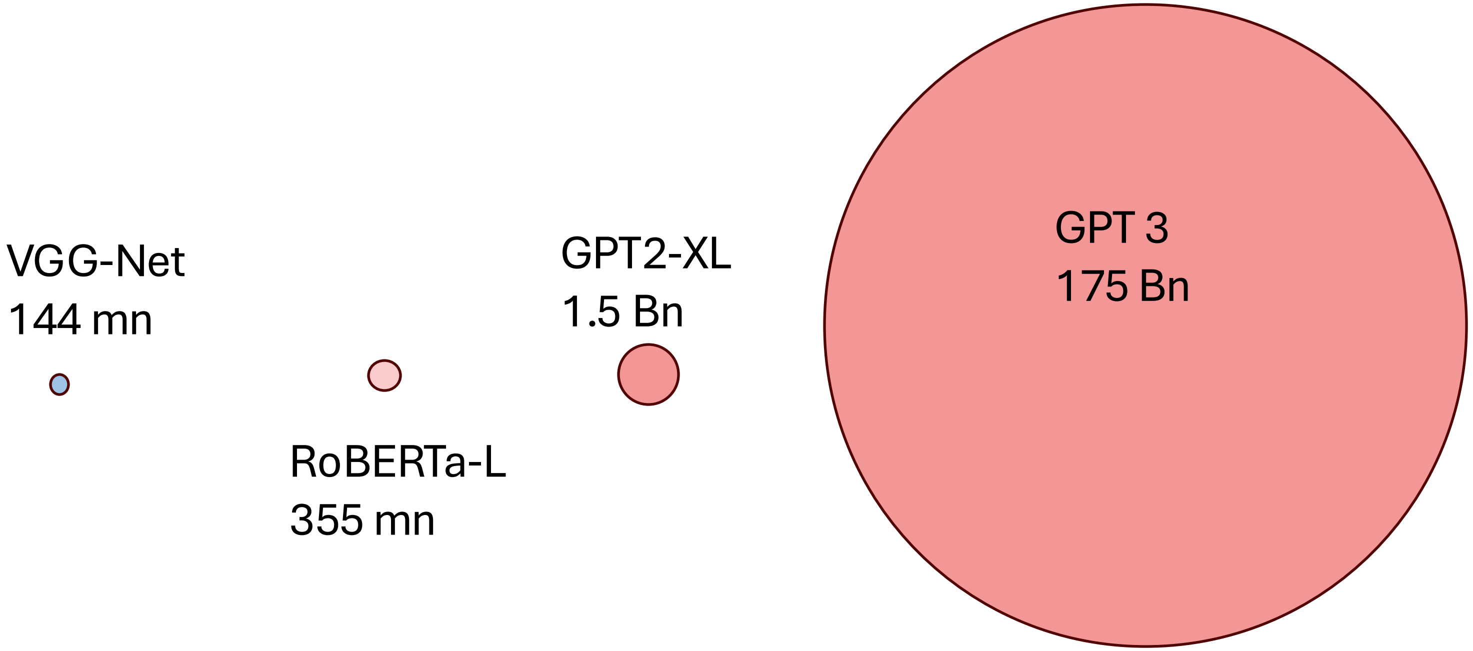


Pretraining
- Pretrain the model on a large dataset.

Finetuning
- Fine-tune of a specific task using a small dataset

Content credits: Chat GPT

Large Language Models



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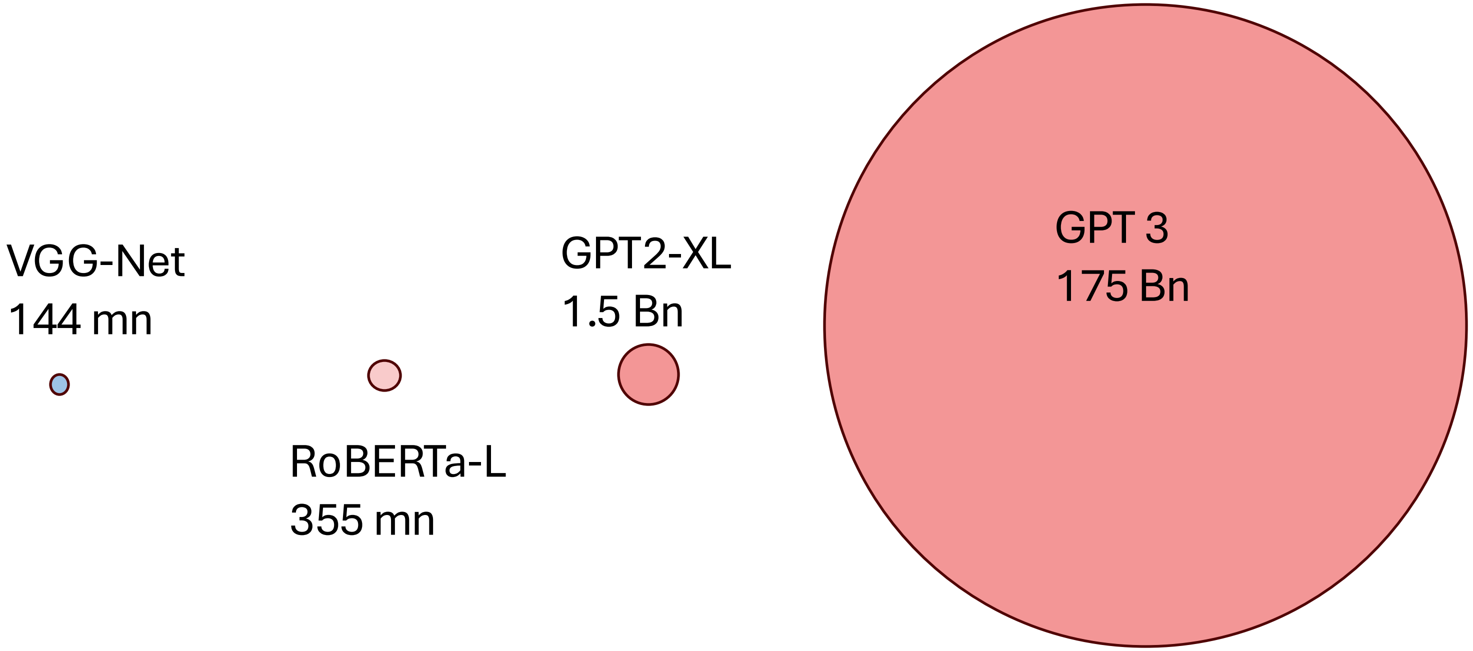
Large Language Models



Model	Size	Approx. Training Time
RoBERTa-L	355 Mn	410 A100-days (410 A100s on 1 day)
GPT-3	175 Bn	60k A100-days (2k A100 on 30 days)

Content credits: https://www.javatpoint.com/nlp

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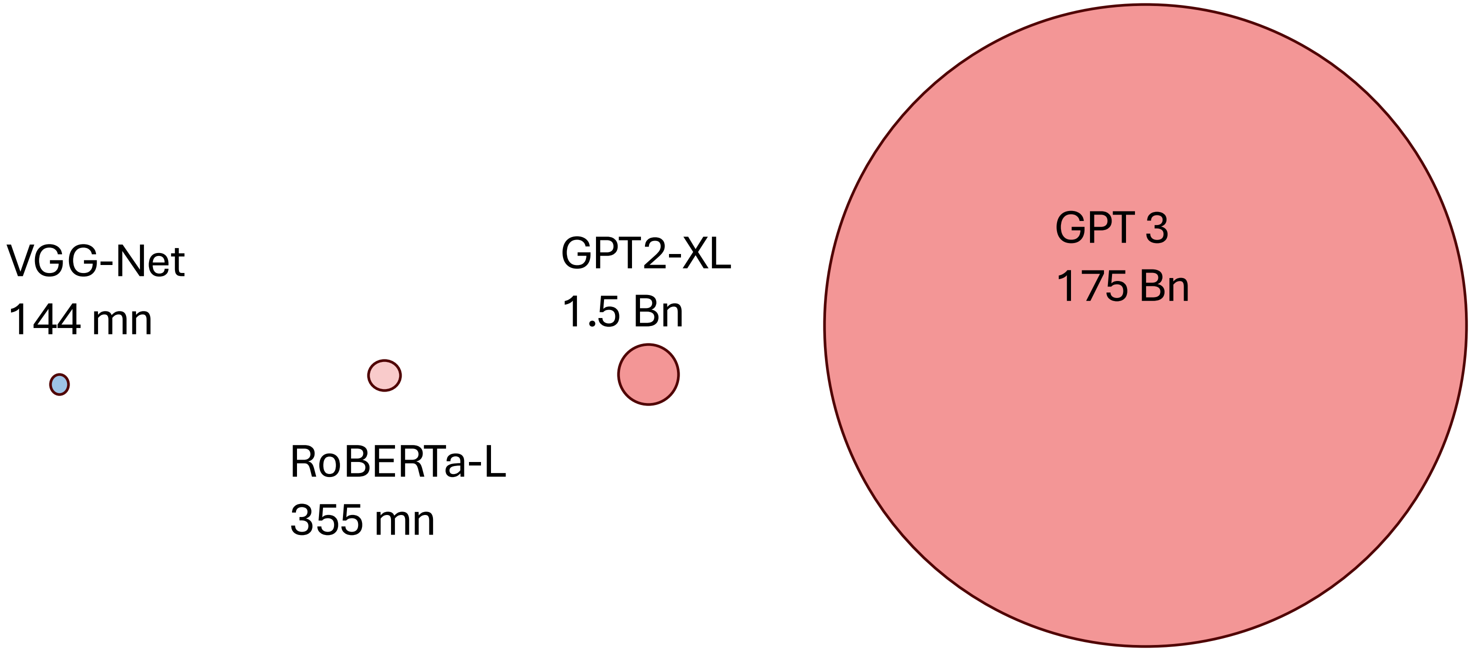

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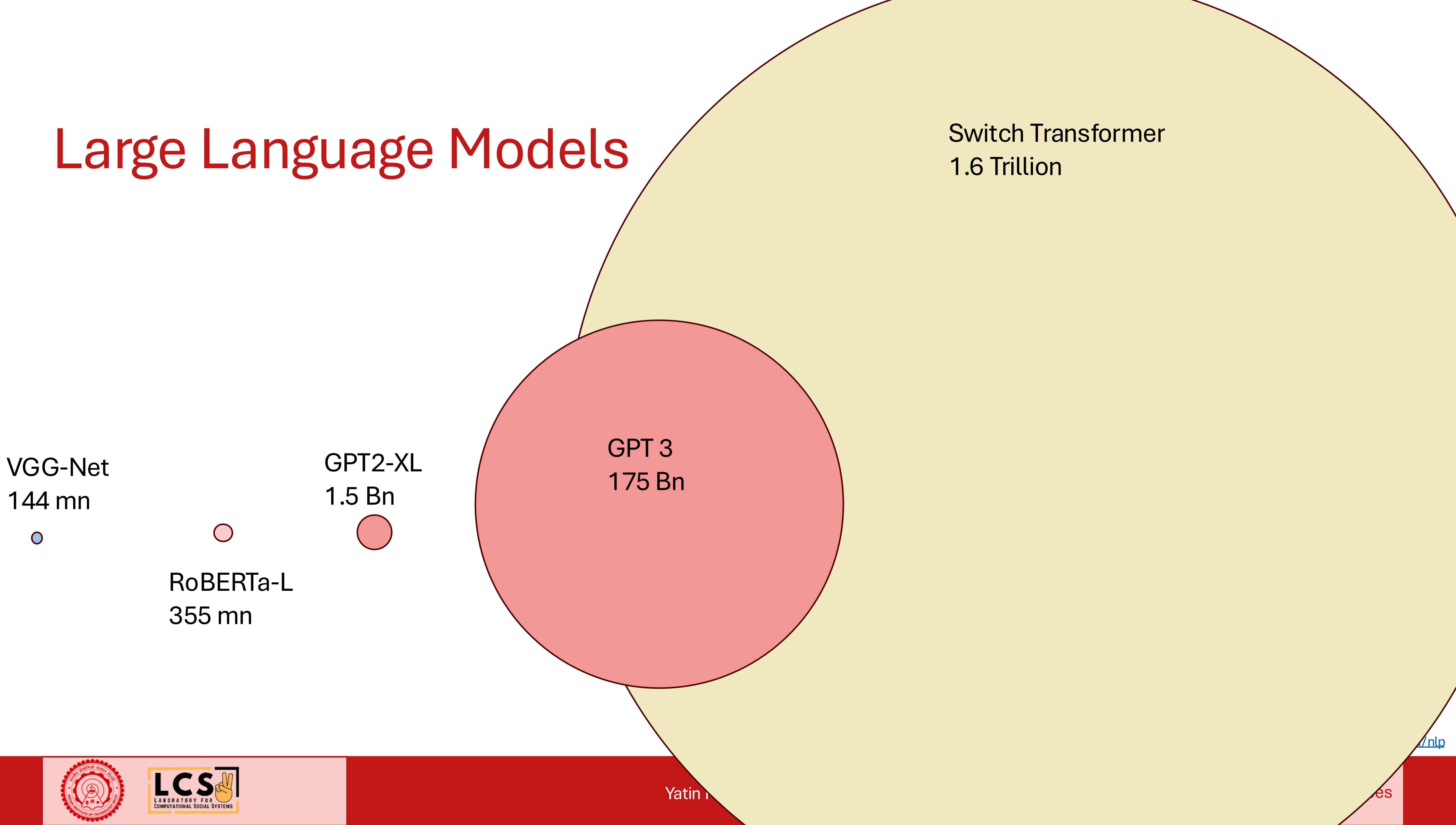

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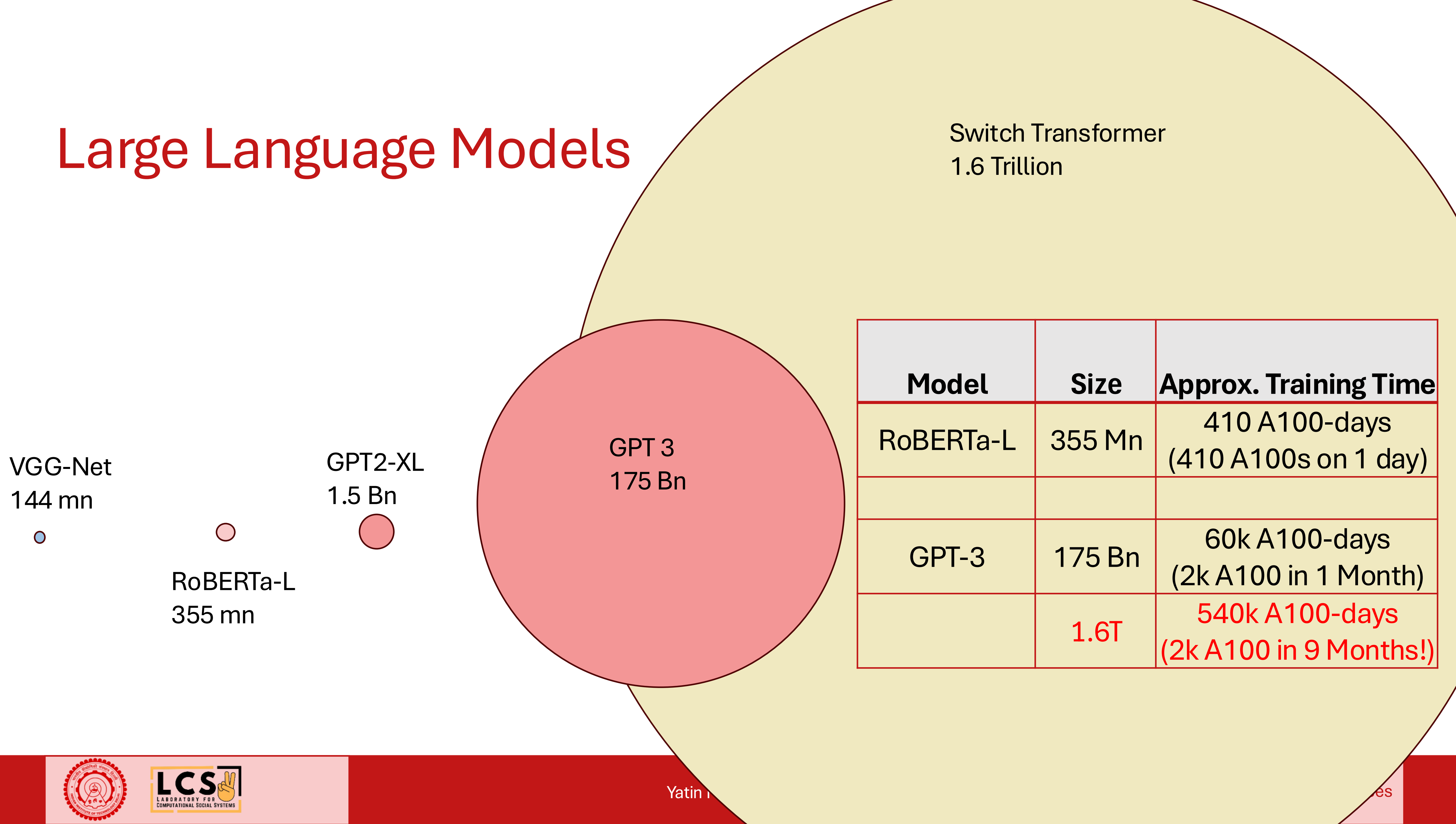

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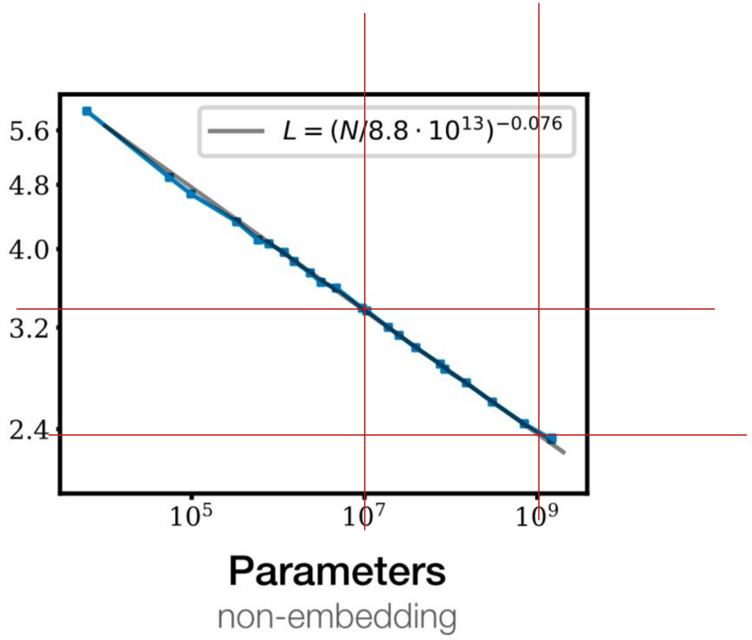
GPT-3	175 Bn	60k A100-days (2k A100 in 1 Month)
	1.6T	540k A100-days (2k A100 in 9 Months!)

Why care about model size?

Content credits: https://www.javatpoint.com/nlp

Neural Scaling Laws

- Performance improve smoothly as we increase the compute, dataset size, or the model size



Source: Scaling Laws for Neural Language Models, Kaplan et al . 2020, Open AI

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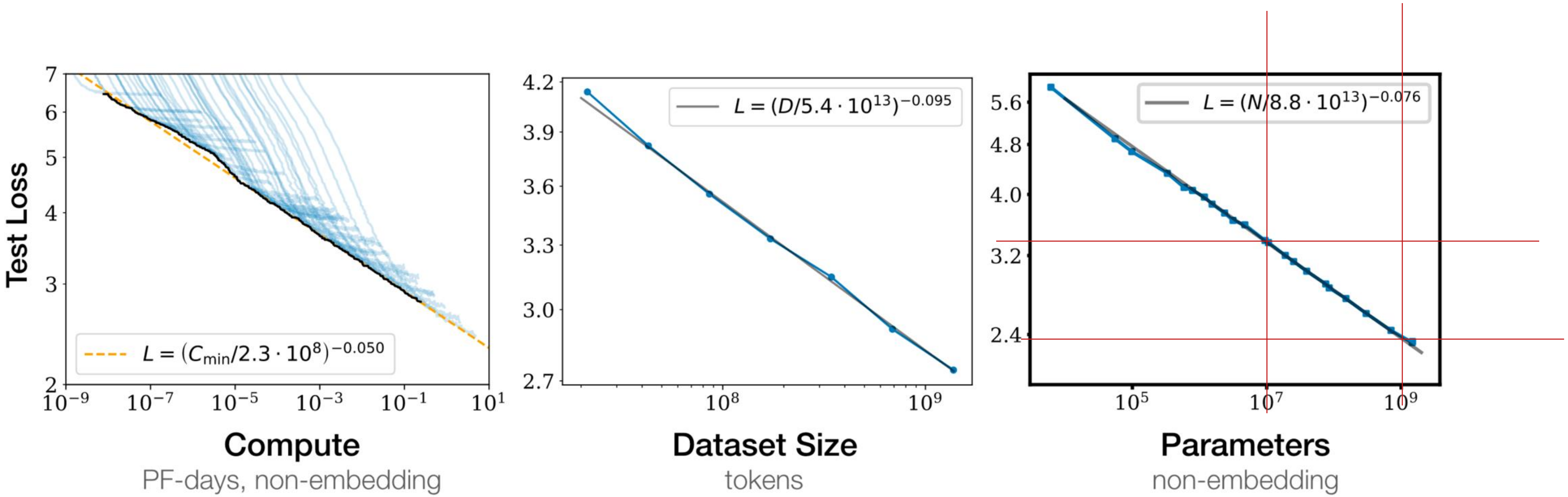

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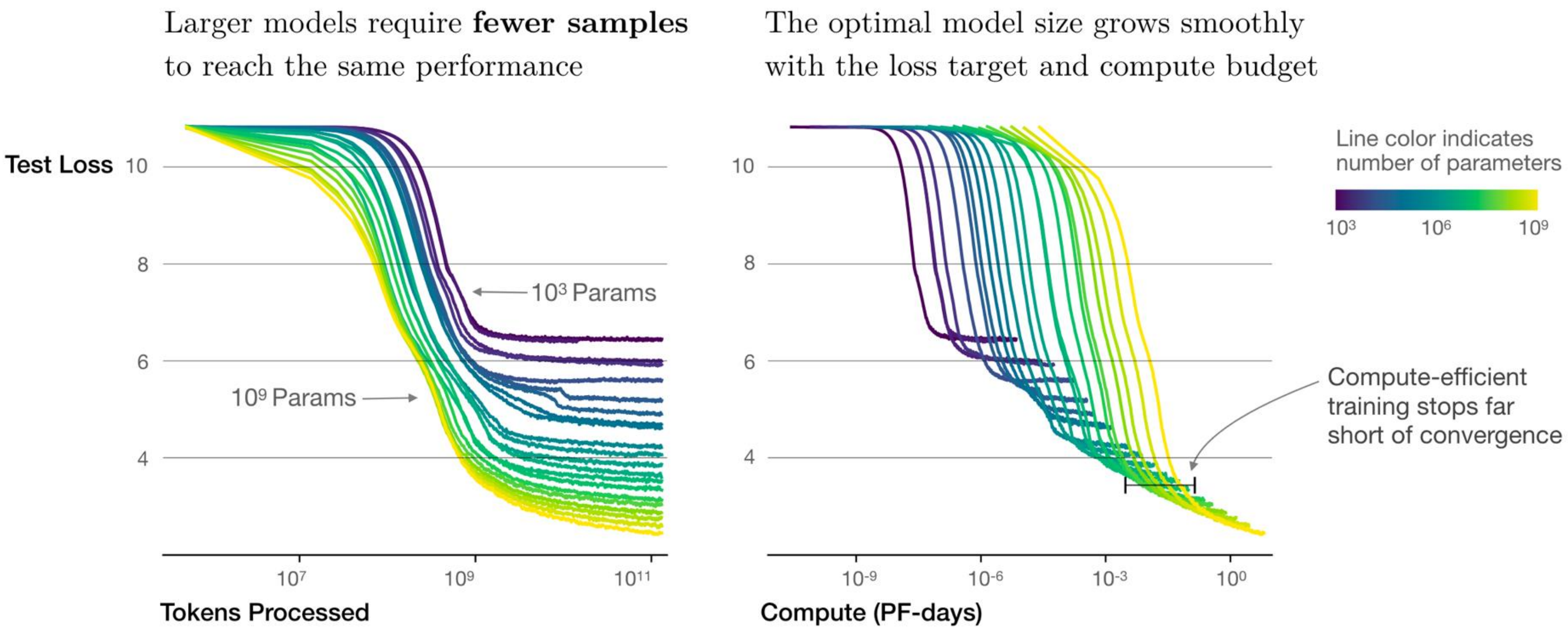
Neural Scaling Laws

- Performance improve smoothly as we increase the compute, dataset size, or the model size
- **Large models are more sample efficient** -- given a fixed computing budget, training a larger model for fewer steps is better than training a smaller model for more steps.



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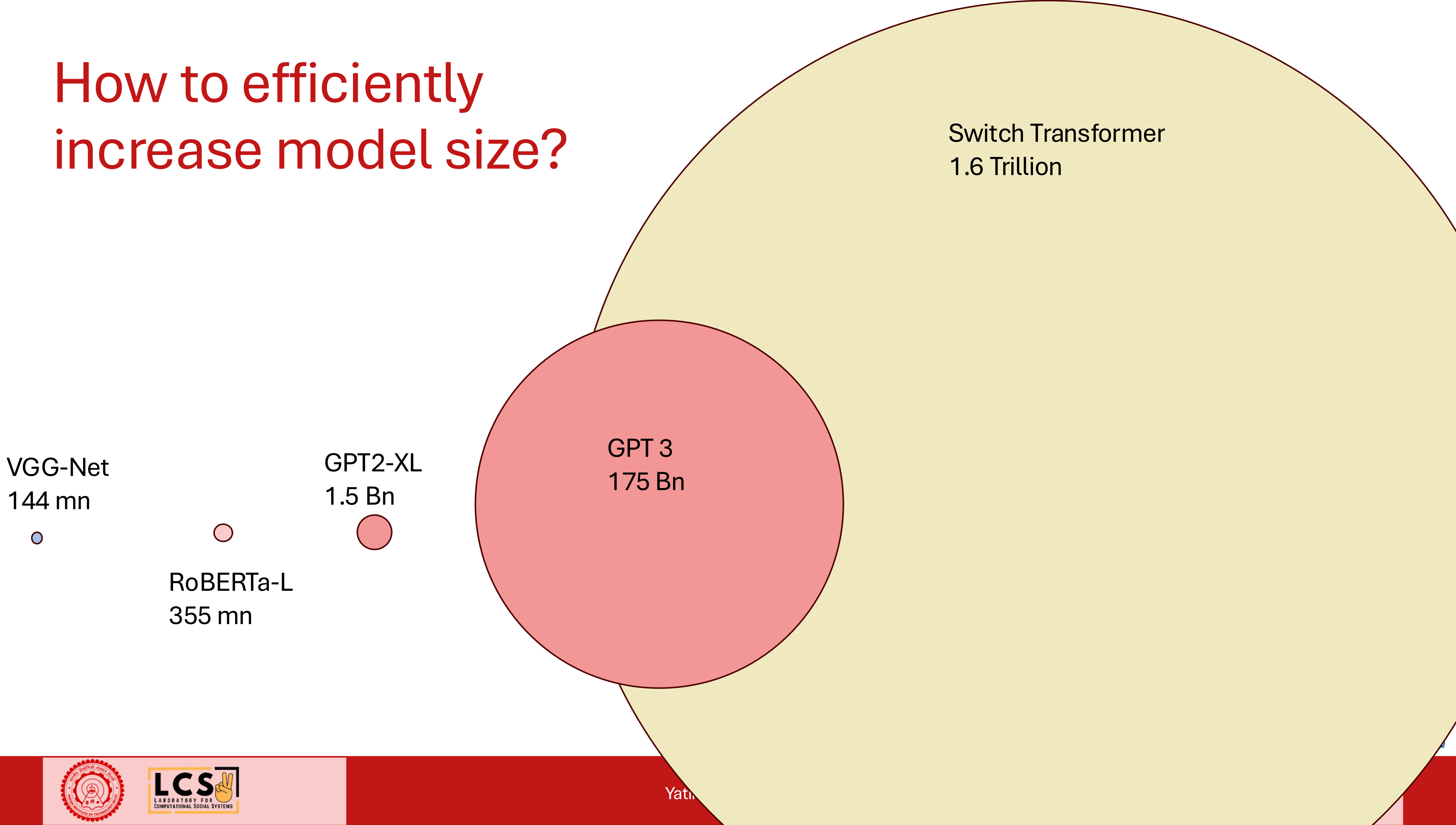
How to efficiently increase model size?



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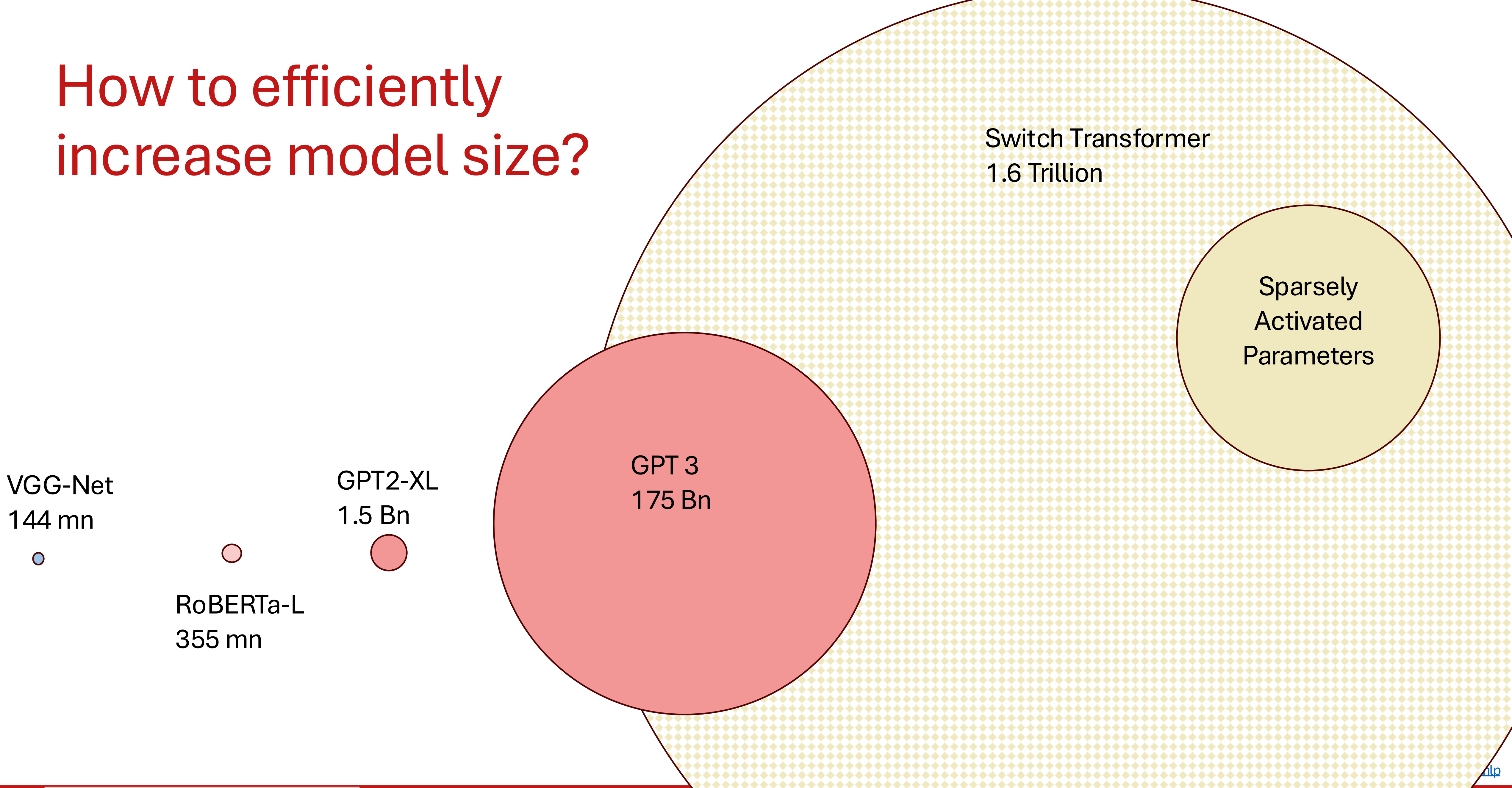


Mixture of Experts

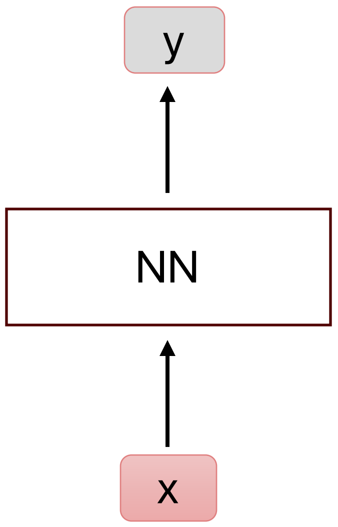
Adaptive Mixtures of Local Experts

Robert A. Jacobs
Michael I. Jordan
Department of Brain and Cognitive Sciences, Massachusetts Institute of Technology, Cambridge, MA 02139 USA

Steven J. Nowlan
Geoffrey E. Hinton
Department of Computer Science, University of Toronto, Toronto, Canada M5S 1A4

We present a new supervised learning procedure for systems composed of many separate networks, each of which learns to handle a subset of the complete set of training cases. The new procedure can be viewed either as a modular version of a multilayer supervised network, or as an associative version of competitive learning. It therefore provides a new link between these two apparently different approaches. We demonstrate that the learning procedure divides up a vowel discrimination task into appropriate subtasks, each of which can be solved by a very simple expert network.

Mixture of Experts



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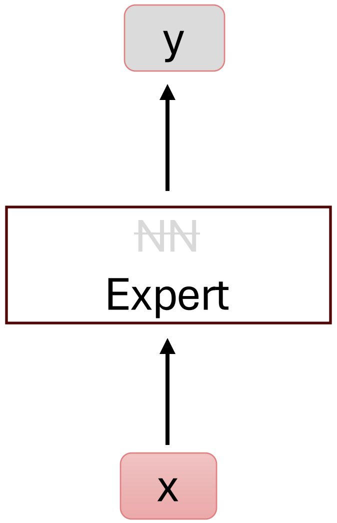

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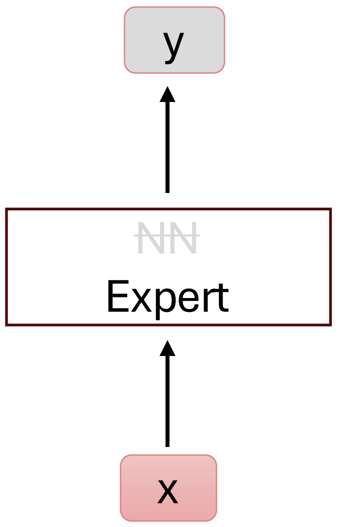

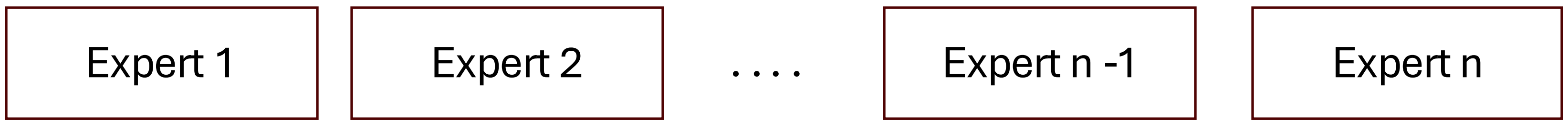

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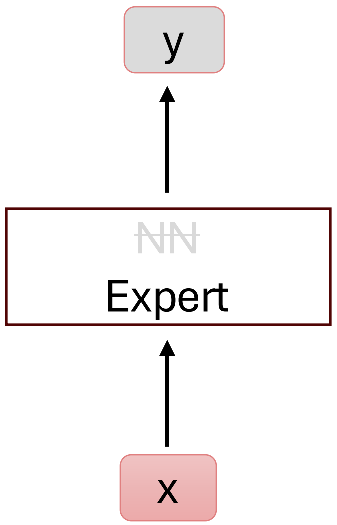

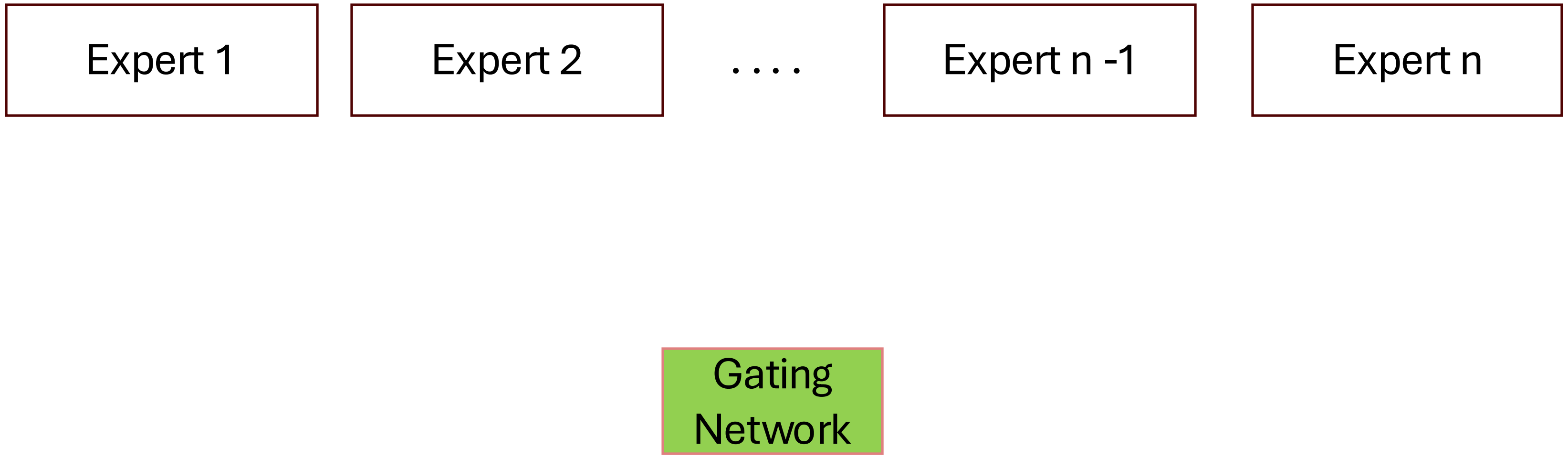

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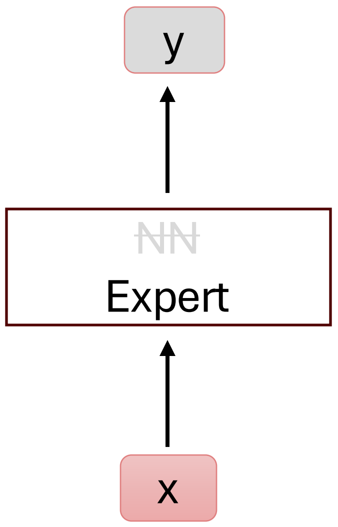

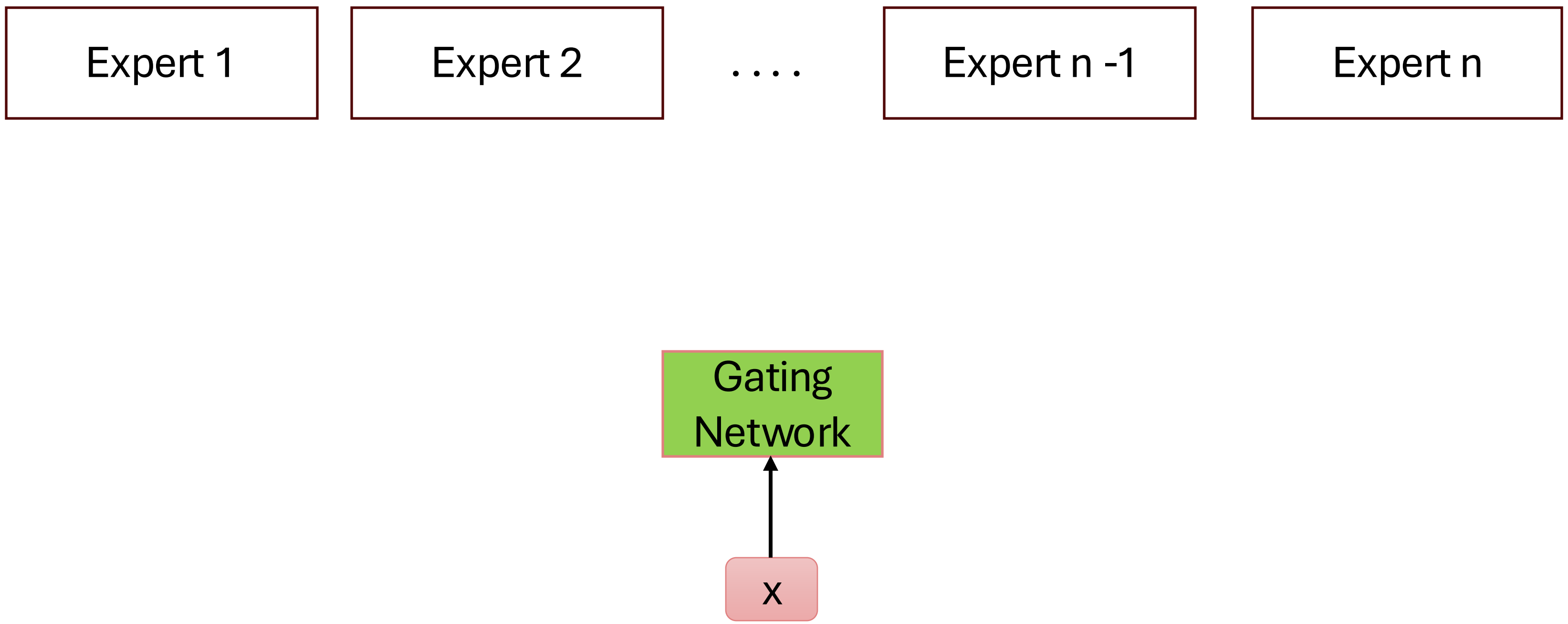

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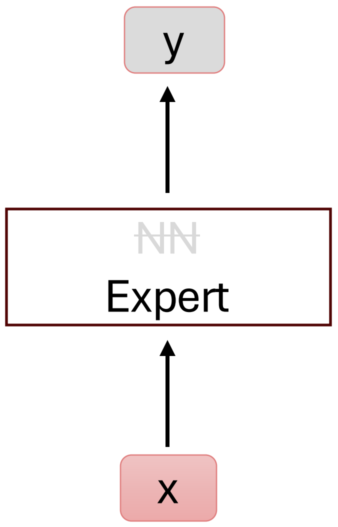

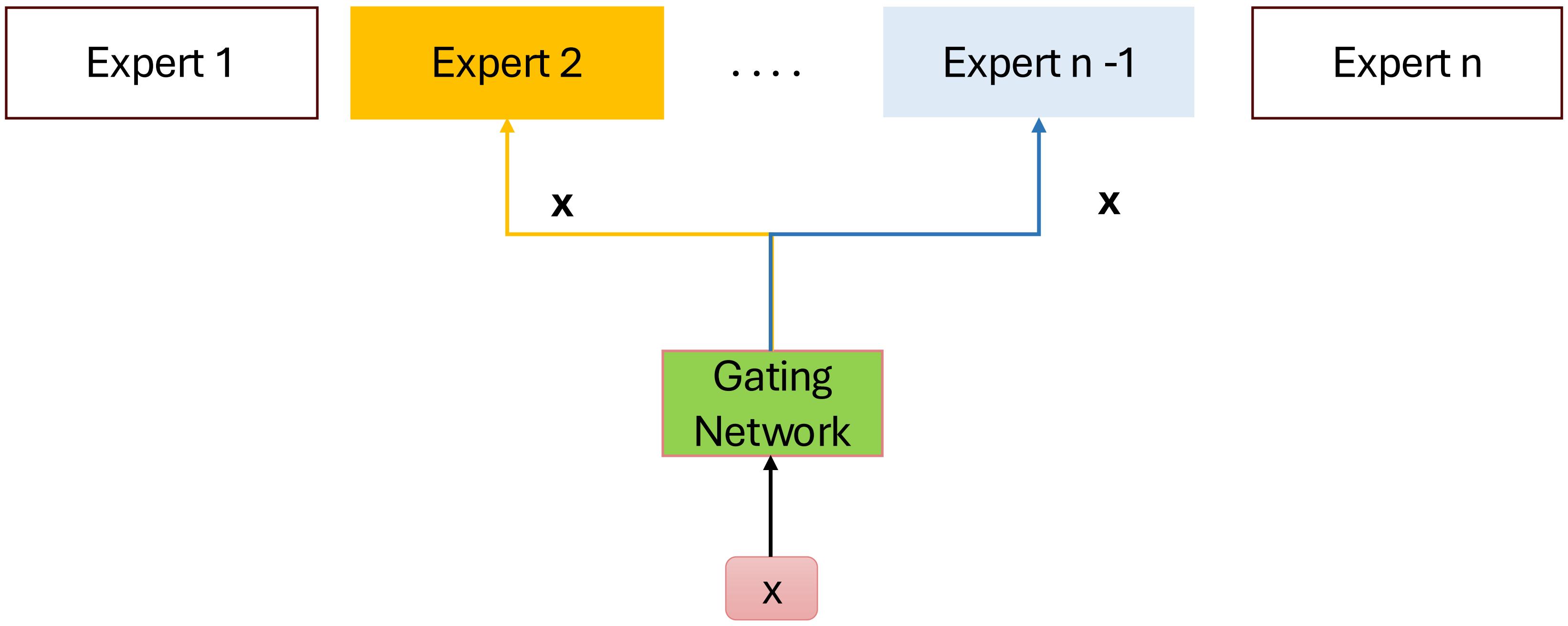

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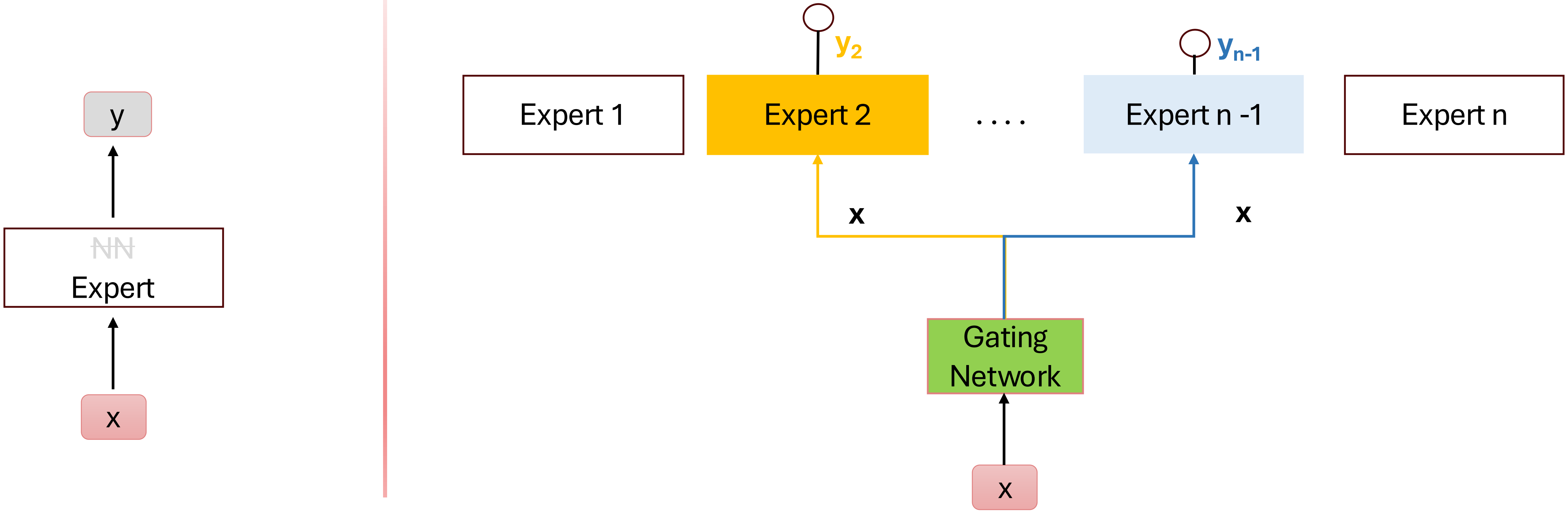

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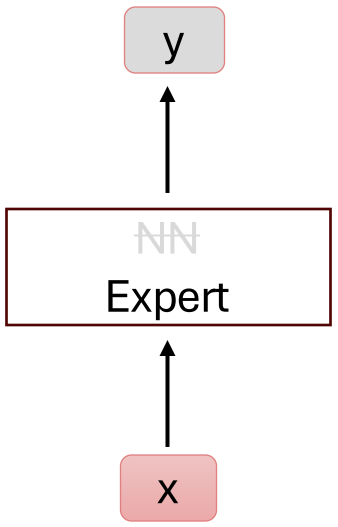

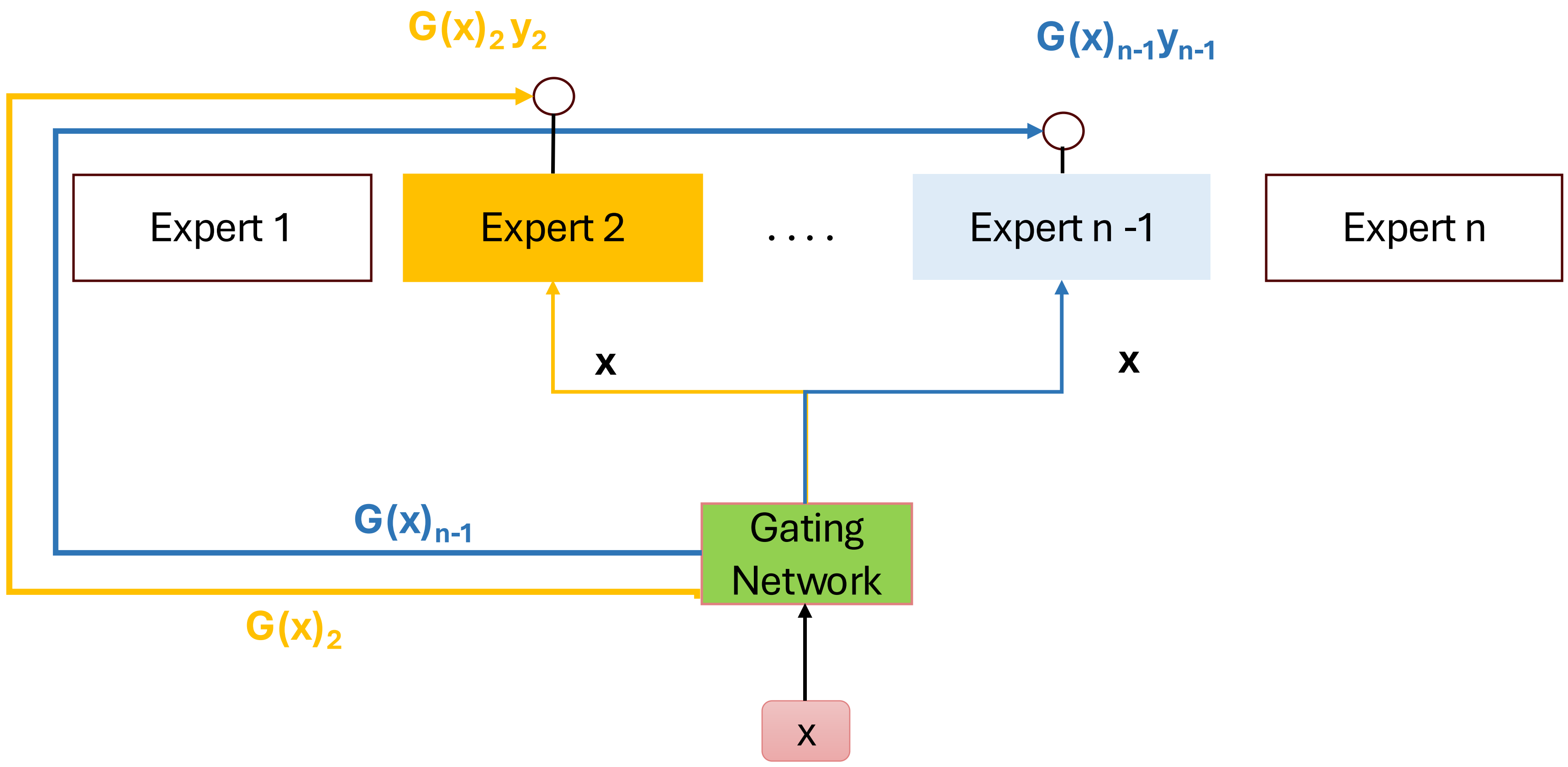

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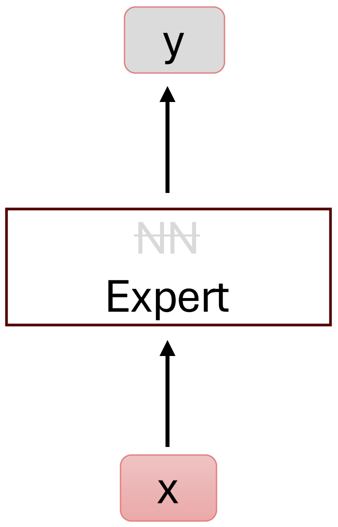
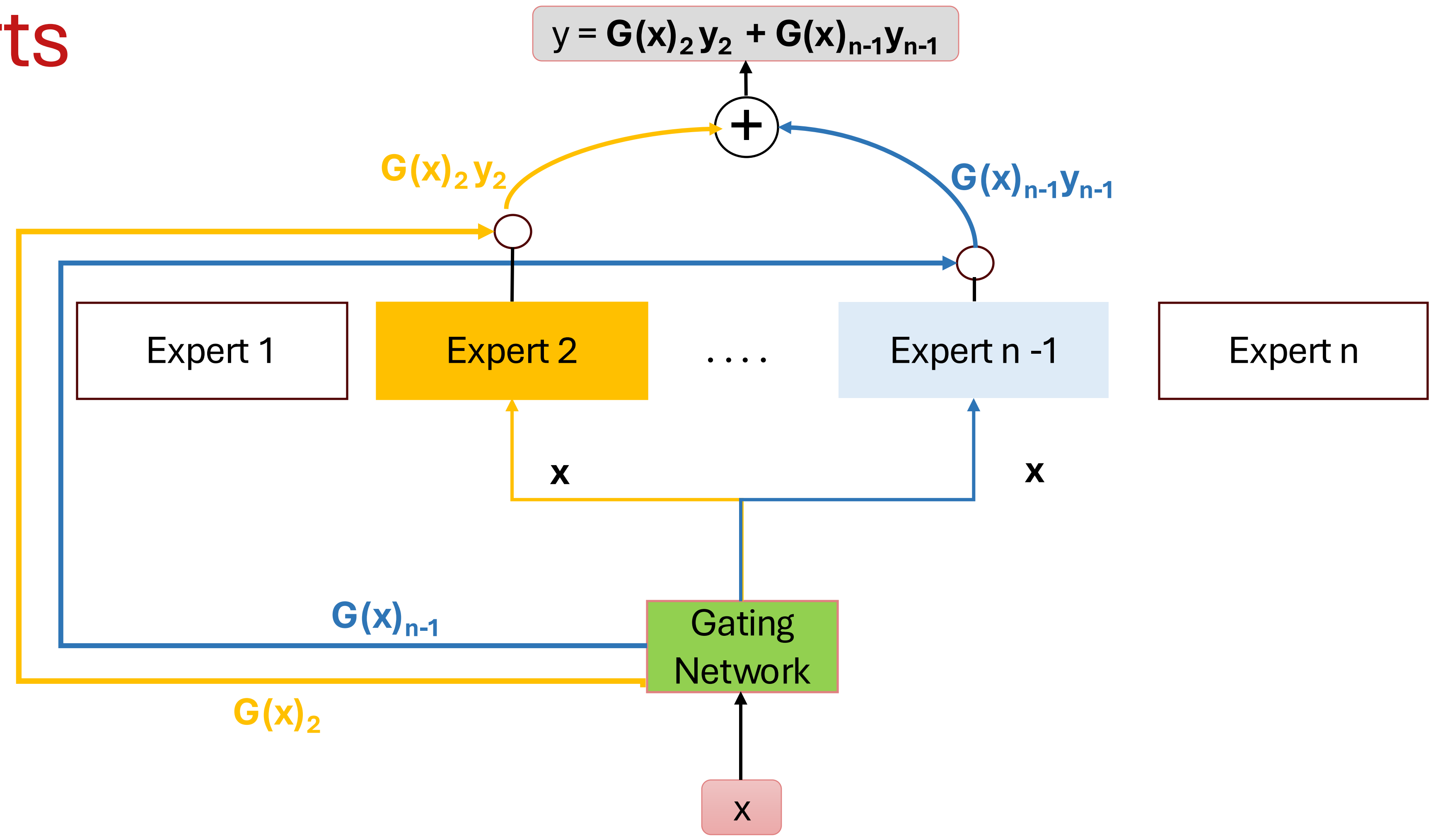
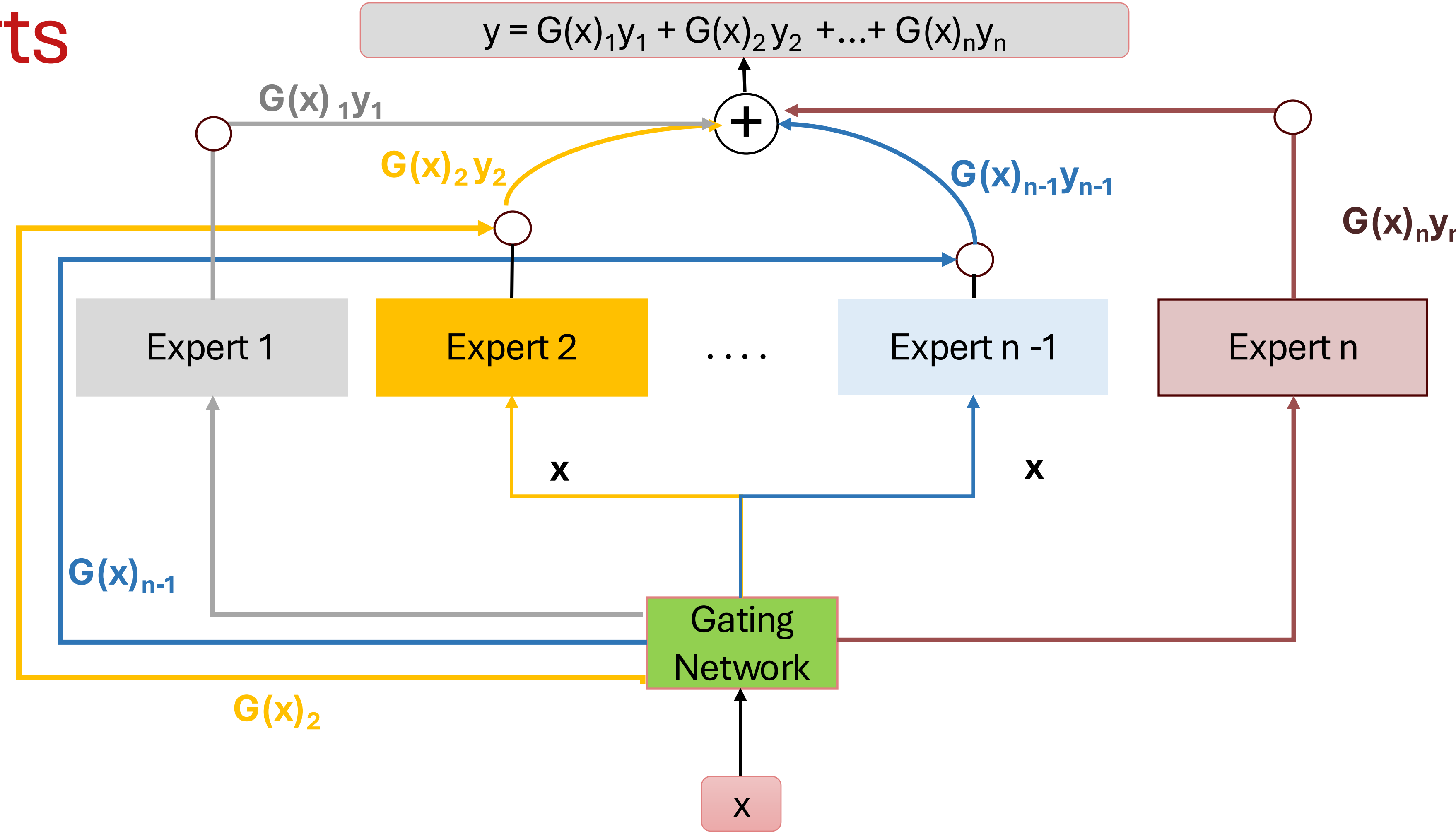




Mixture of Experts

$$G_\sigma(x) = \text{Softmax}(x \cdot W_g)$$



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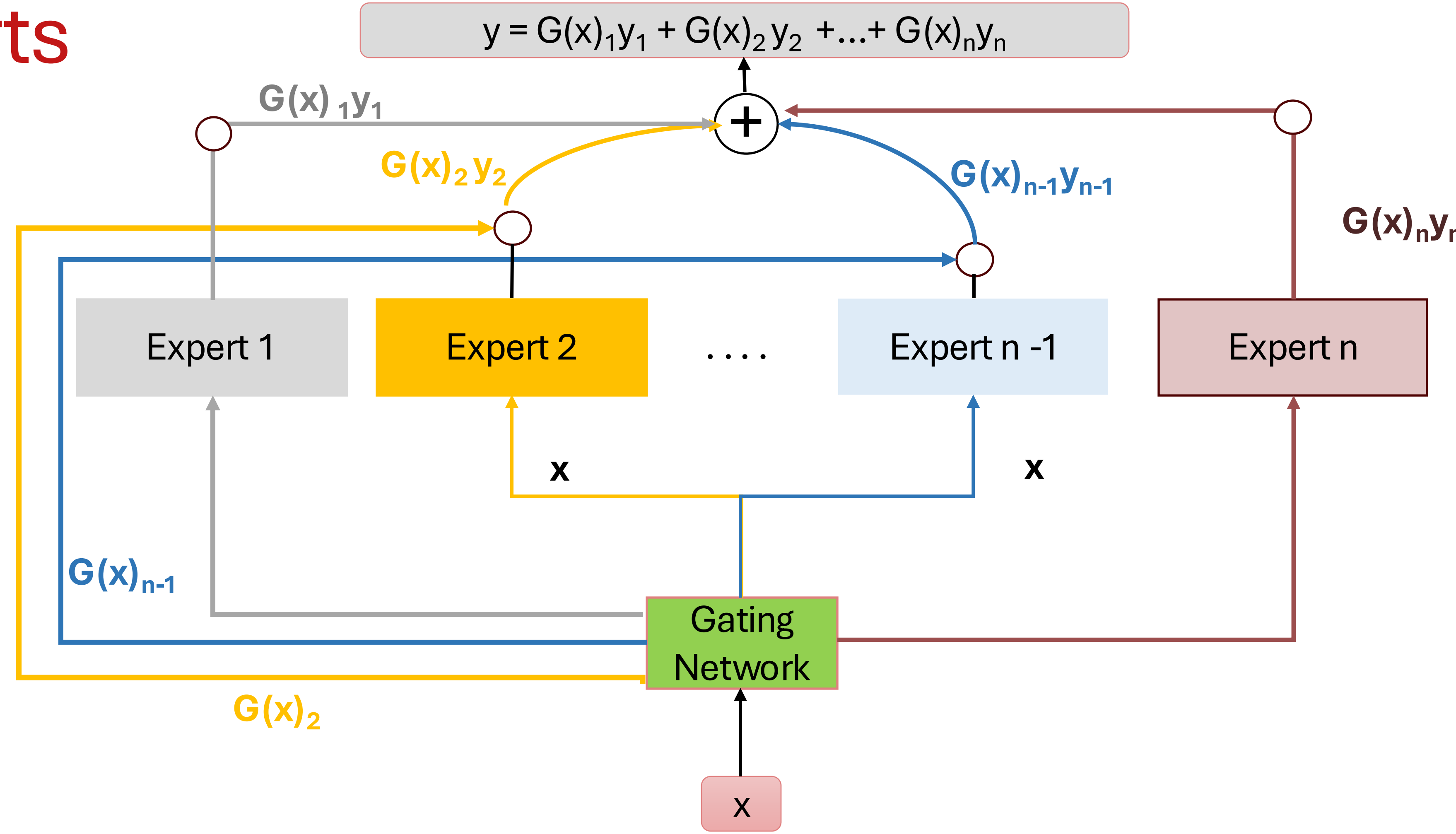

Mixture of Experts - Chronology

- Mixture of Experts Model [Jacobs et al., 1991; Jordan and Jacobs, 1994; Jordan et al., 1997; Tresp, 2001; Collobert et al., 2002;]

Content credits: https://www.youtube.com/watch?v=TwHPxUAuqy4

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- Mixtral-8x7B [Jiang et al. 2024] - Apache 2.0 license; surpasses GPT-3.5 Turbo, Claude-2.1, Gemini Pro, and Llama 2 70B - chat model on human benchmarks

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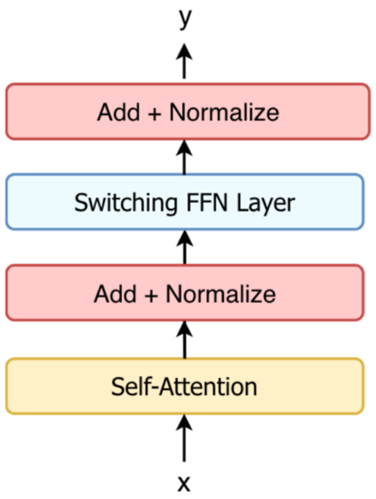
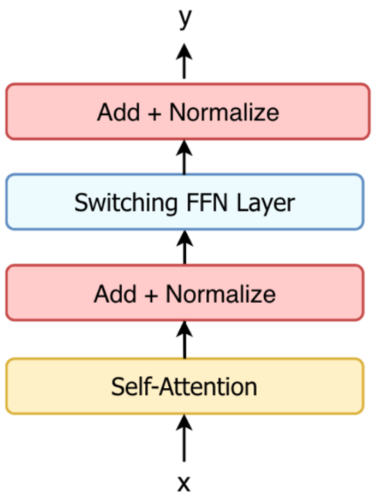
Mixture of Experts as a Layer



Content credits: Switch Transformers: Scaling to Trillion Parameter Models with Simple and Efficient Sparsity

Mixture of Experts as a Layer



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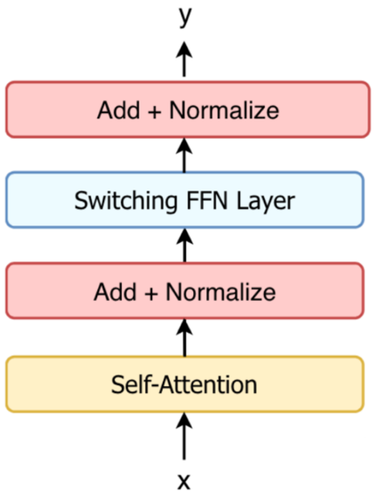

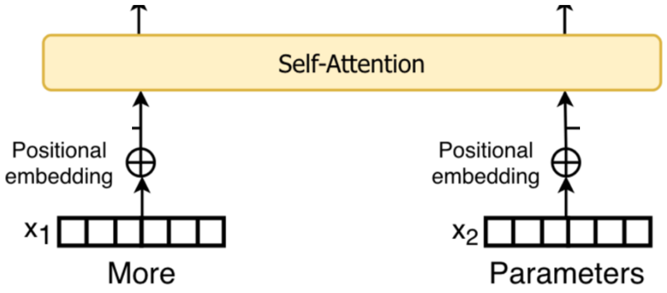

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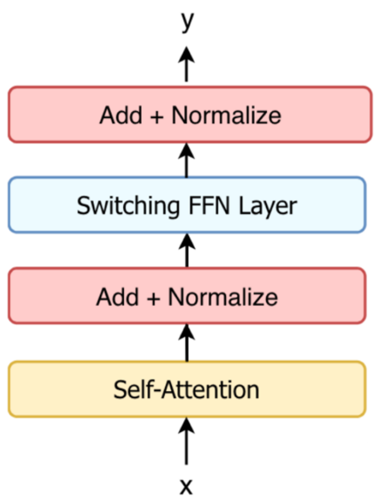

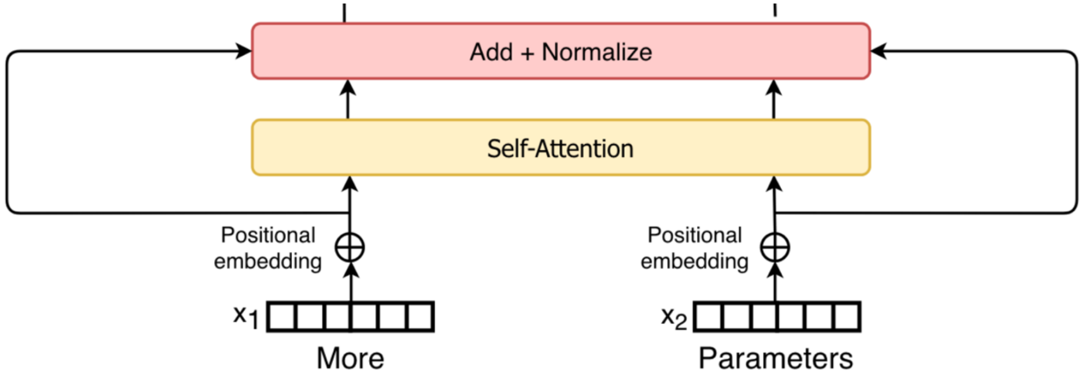

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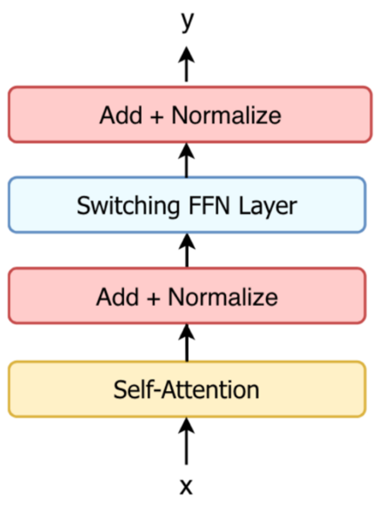

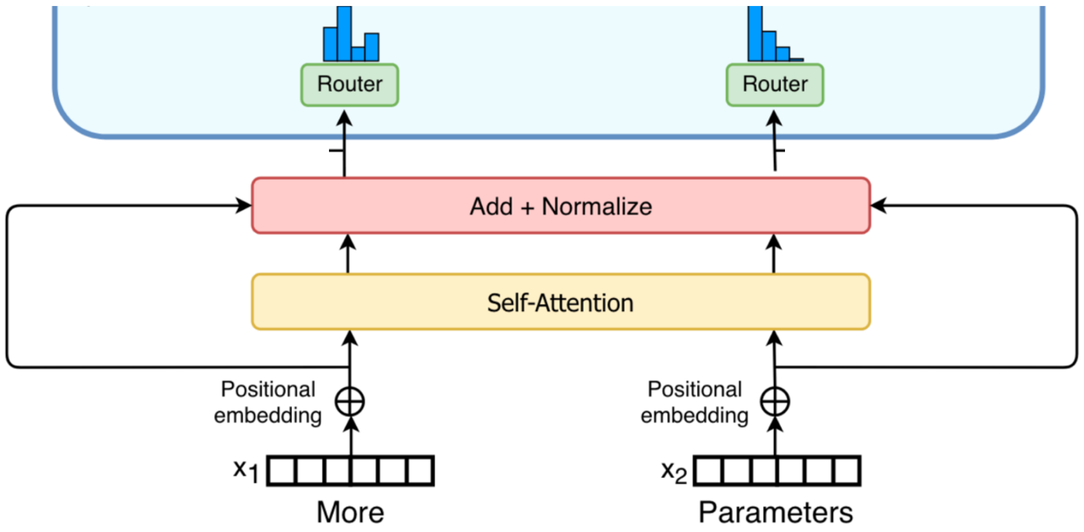

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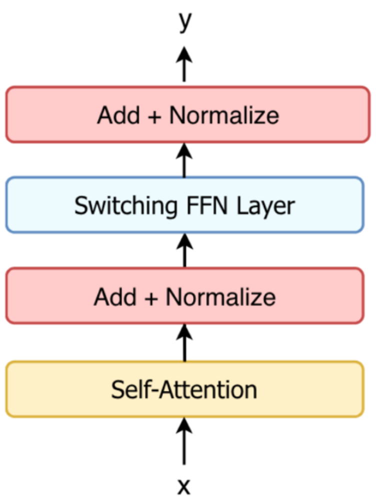

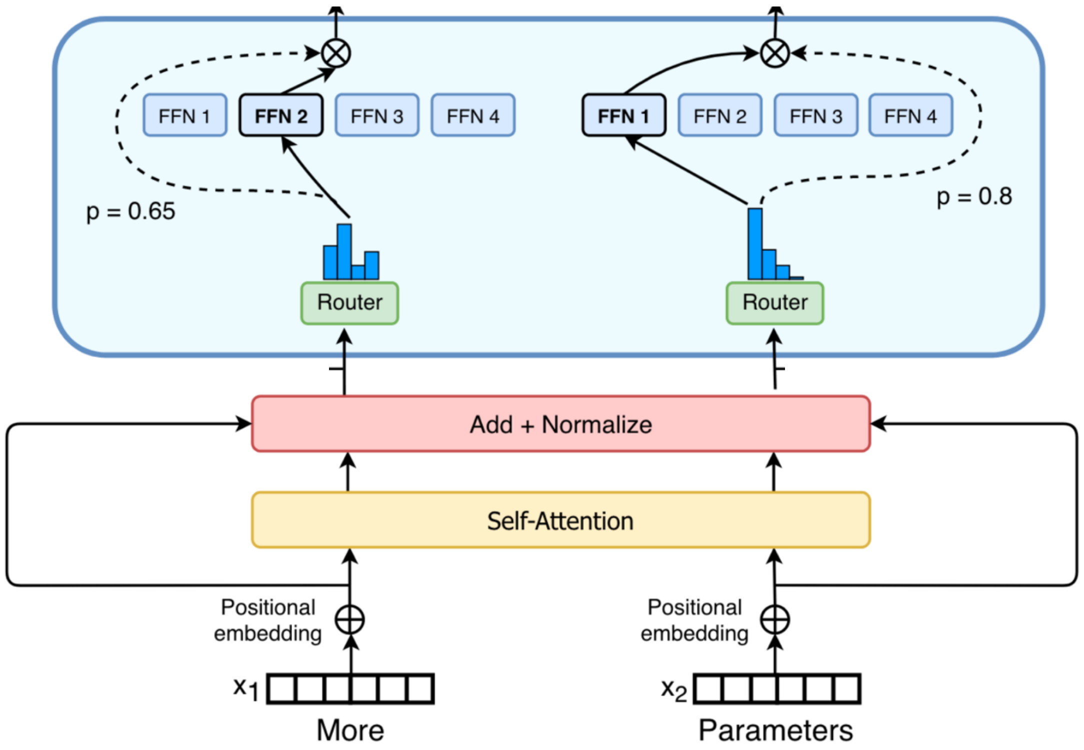

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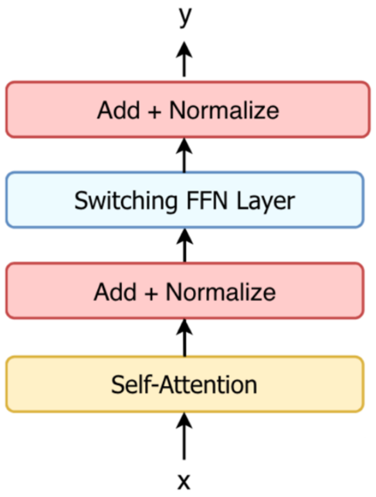

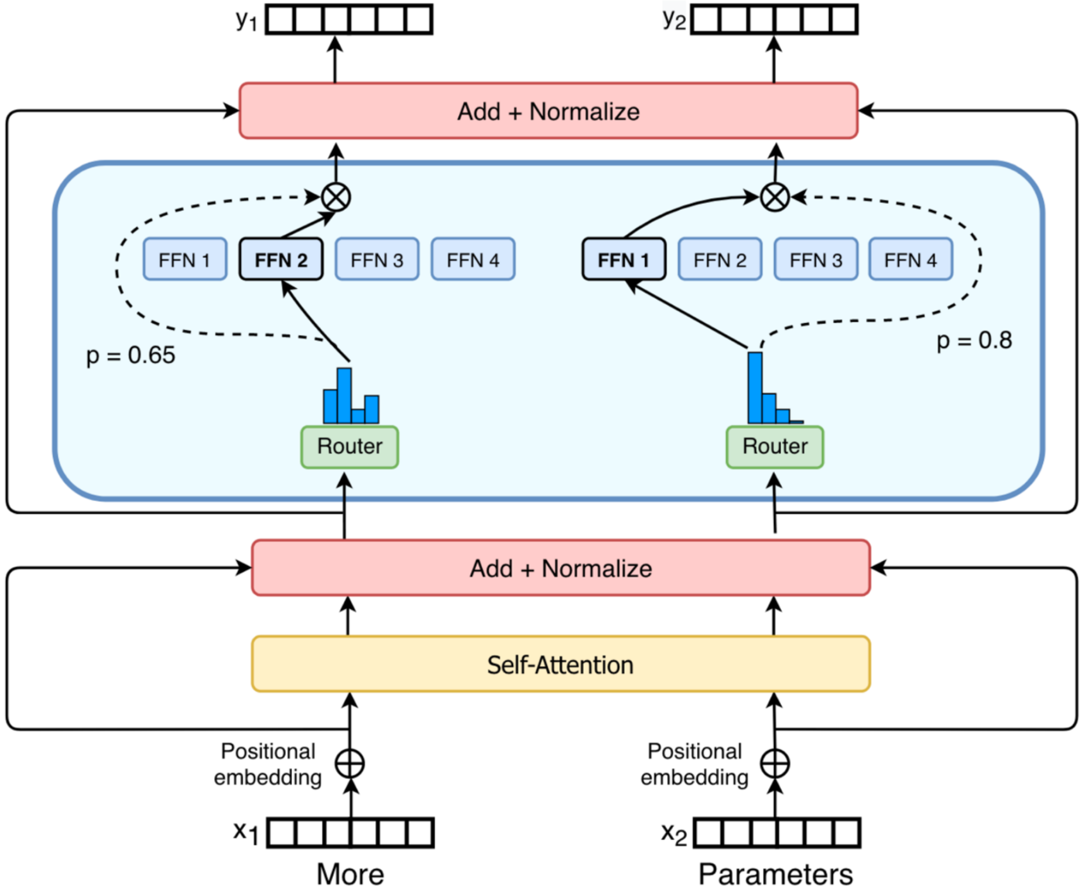

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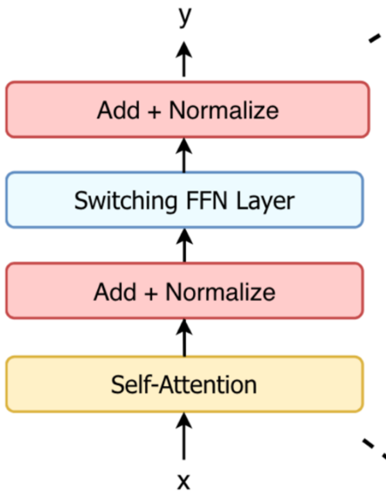
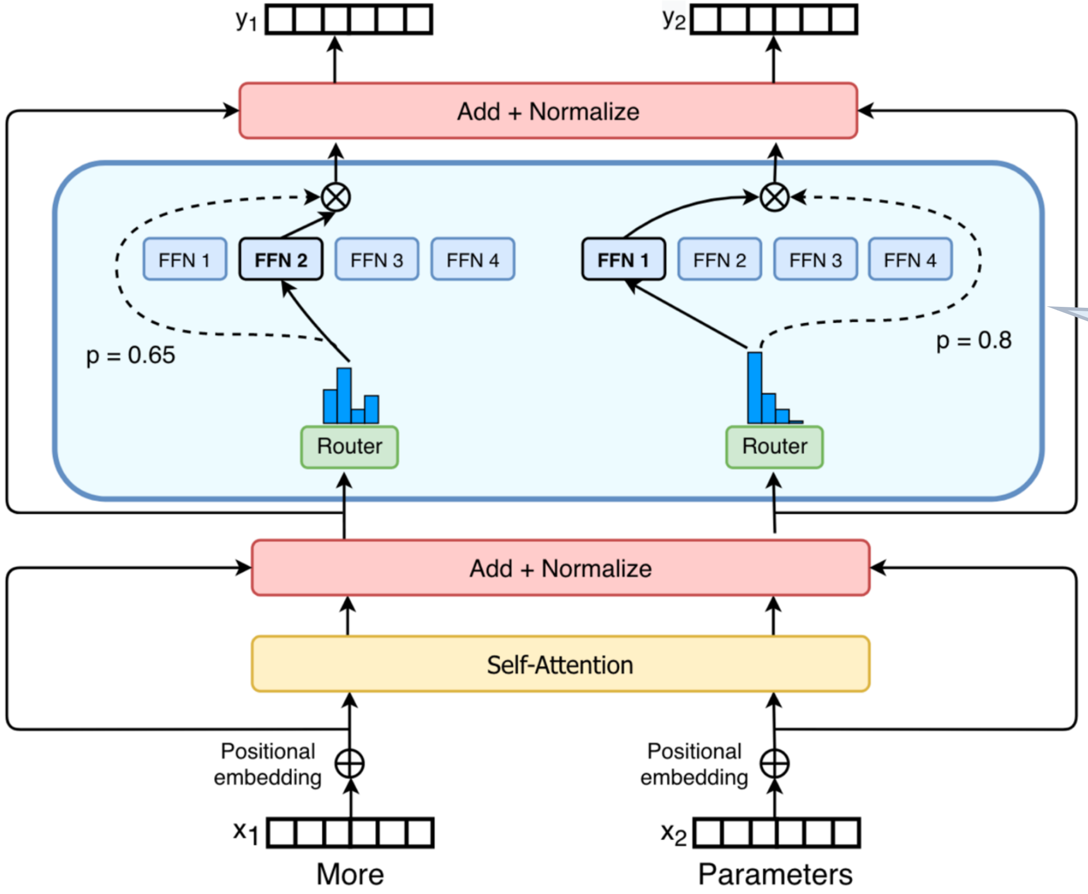


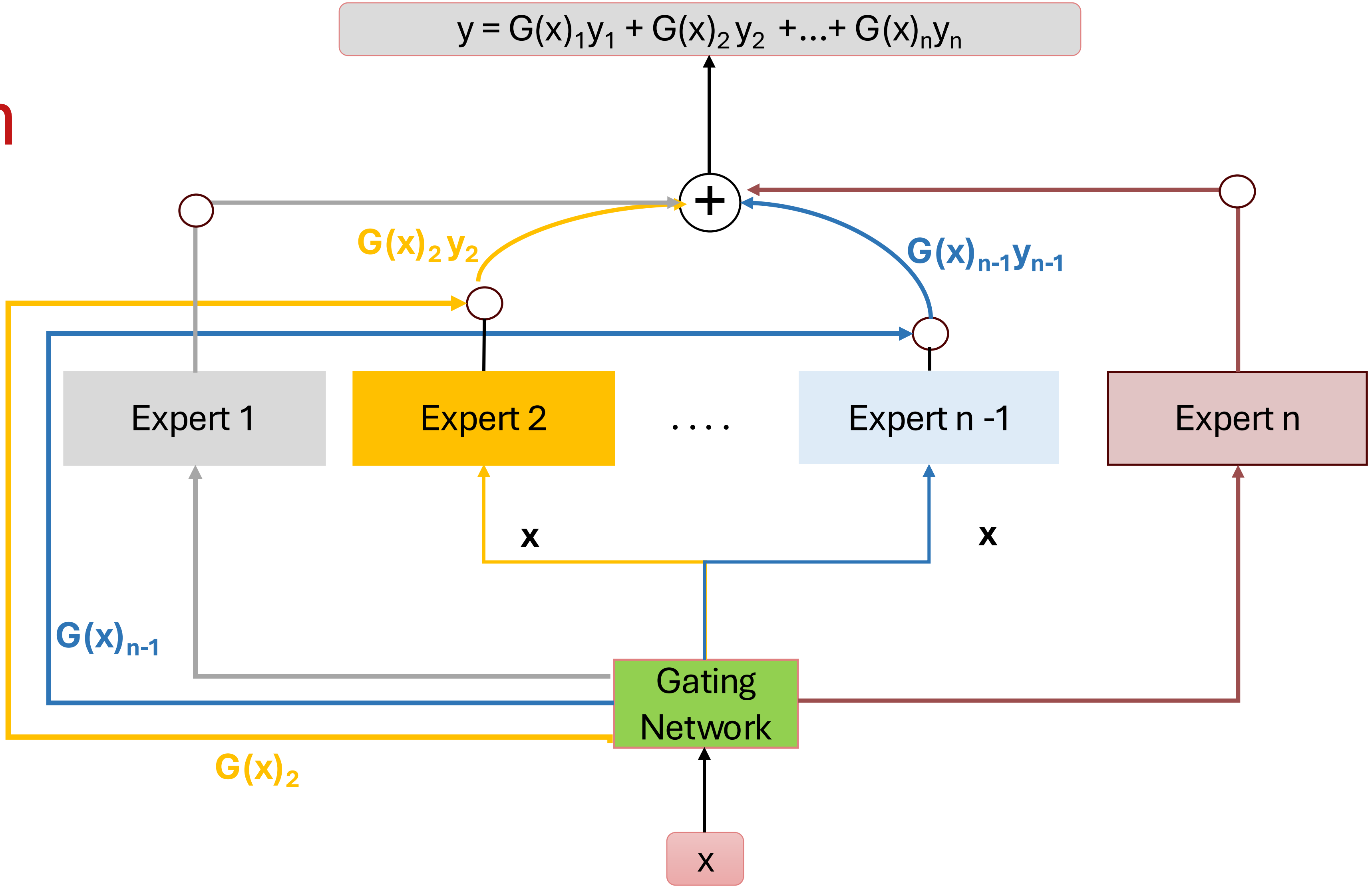
Dense Transformer Block



MoE Layer

Sparsely Activated MoE Transformer Block

MoE Layer Backpropagation



MoE Layer Backpropagation

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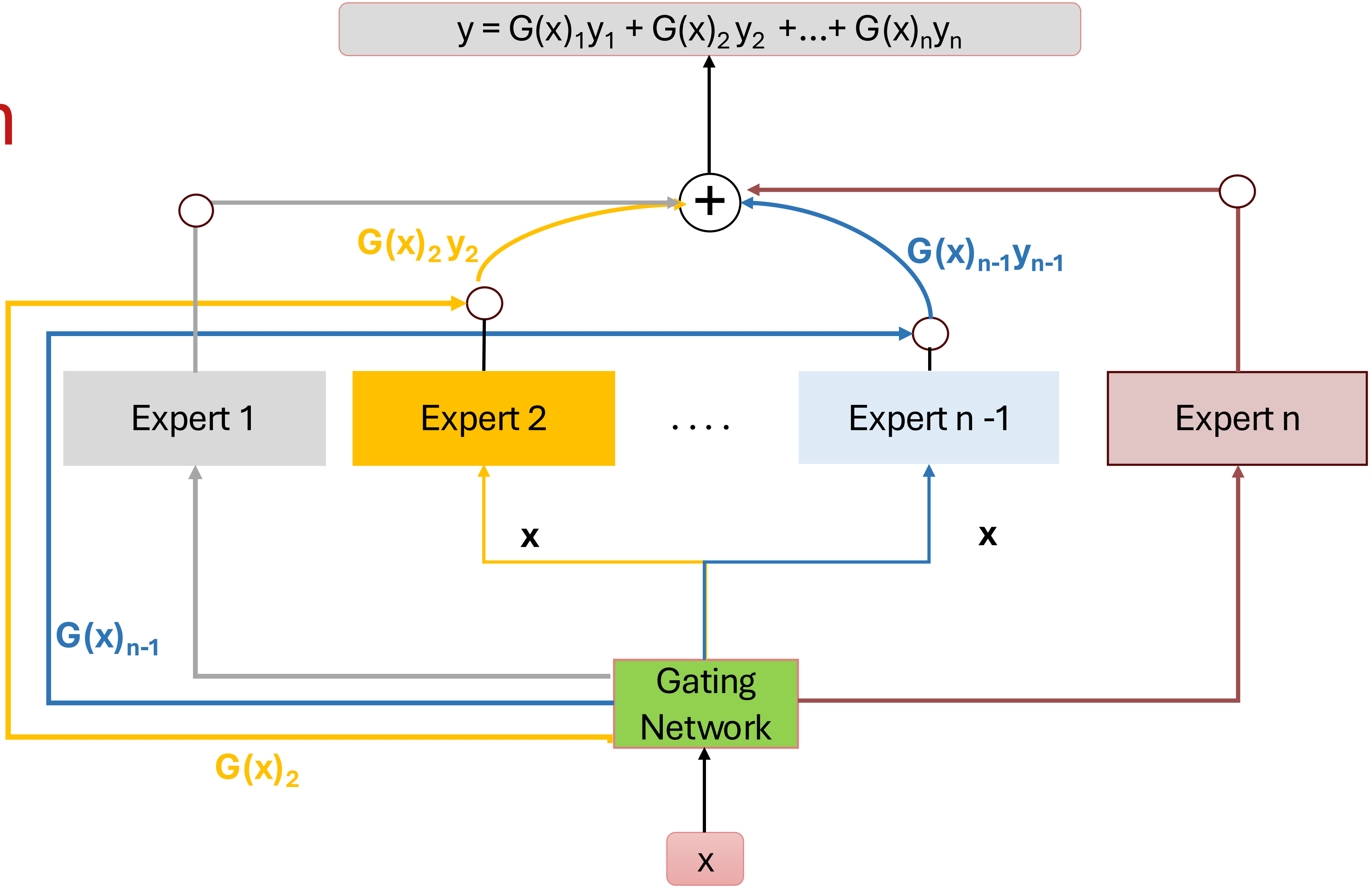

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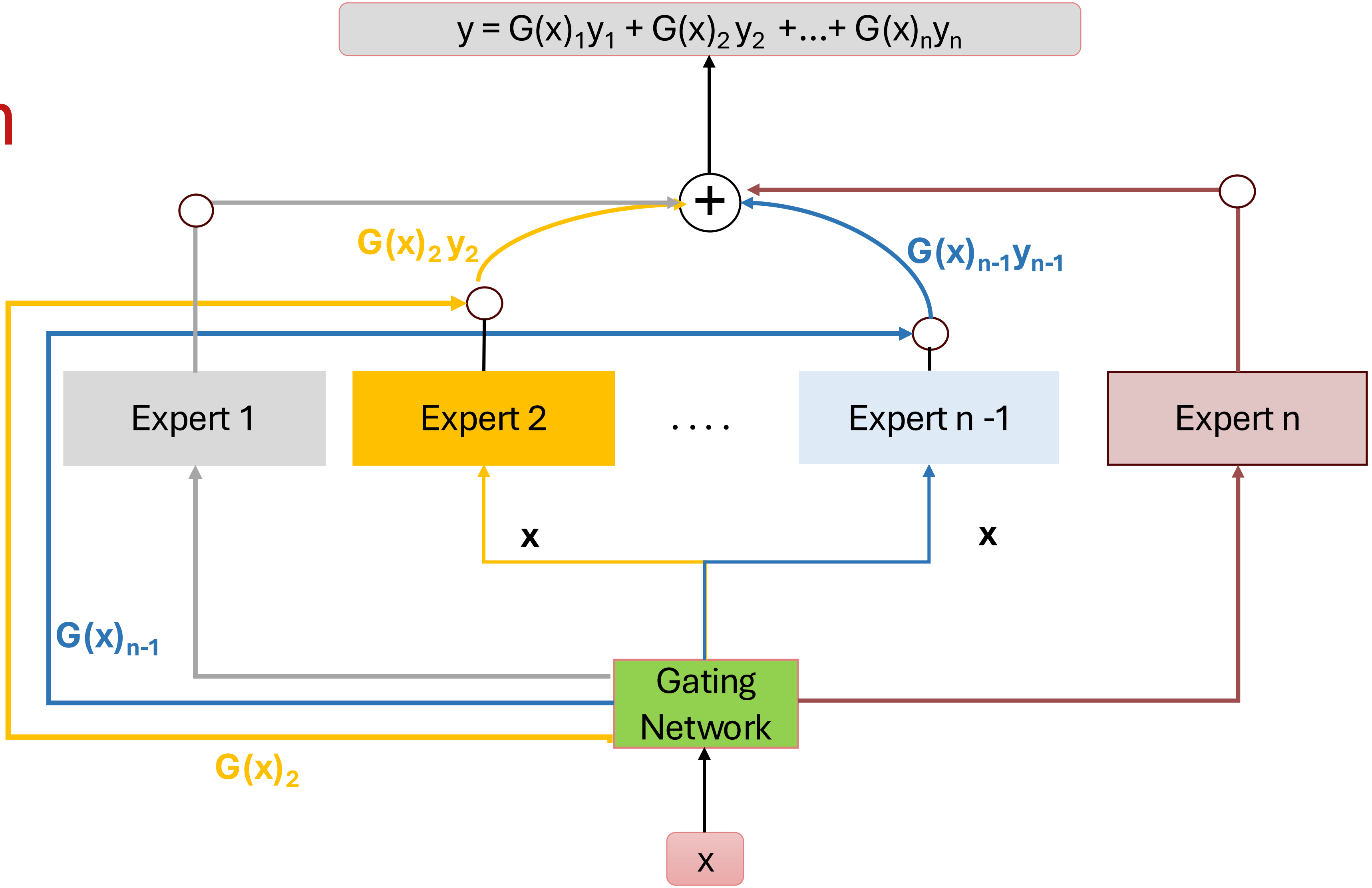

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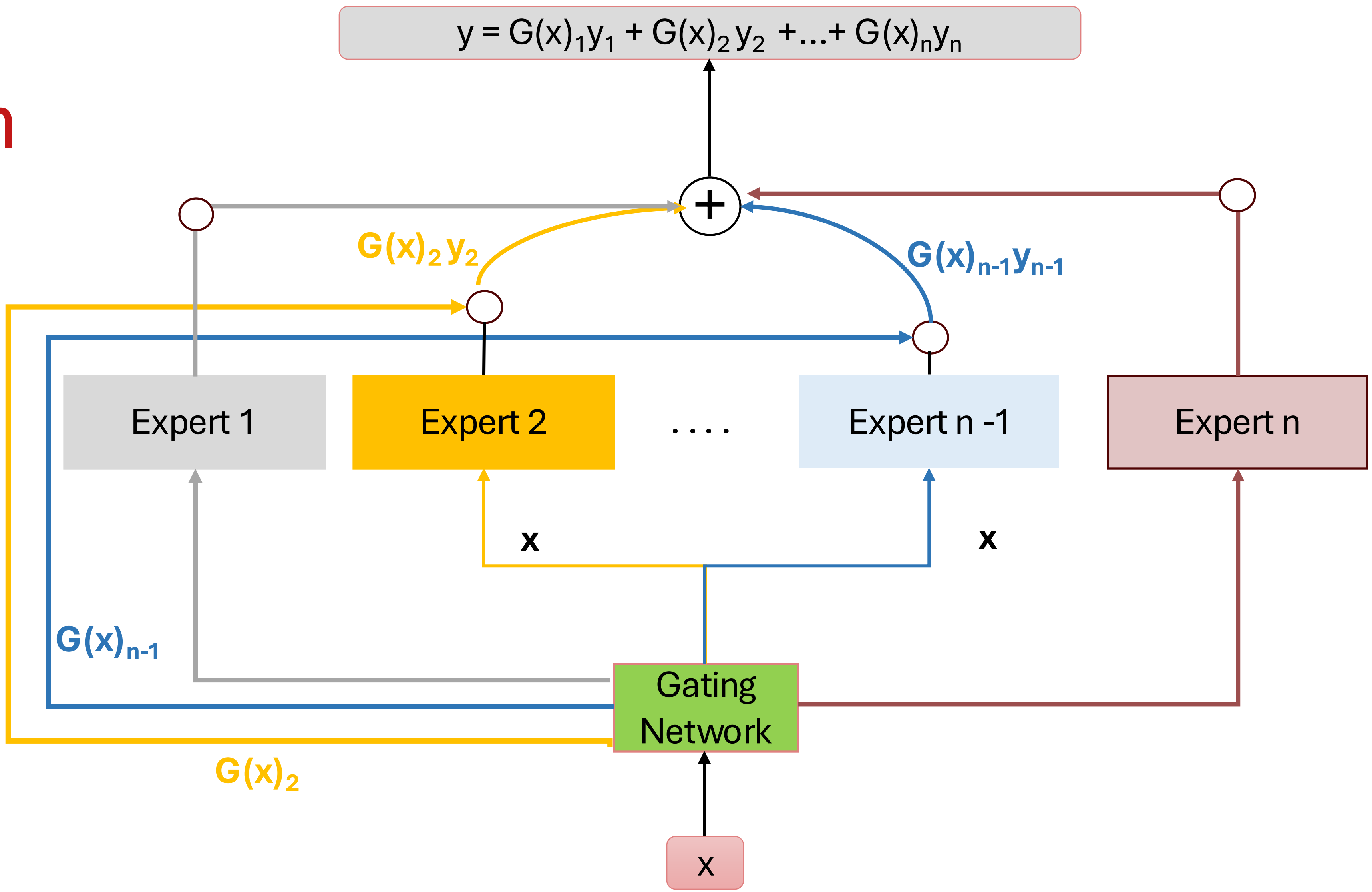

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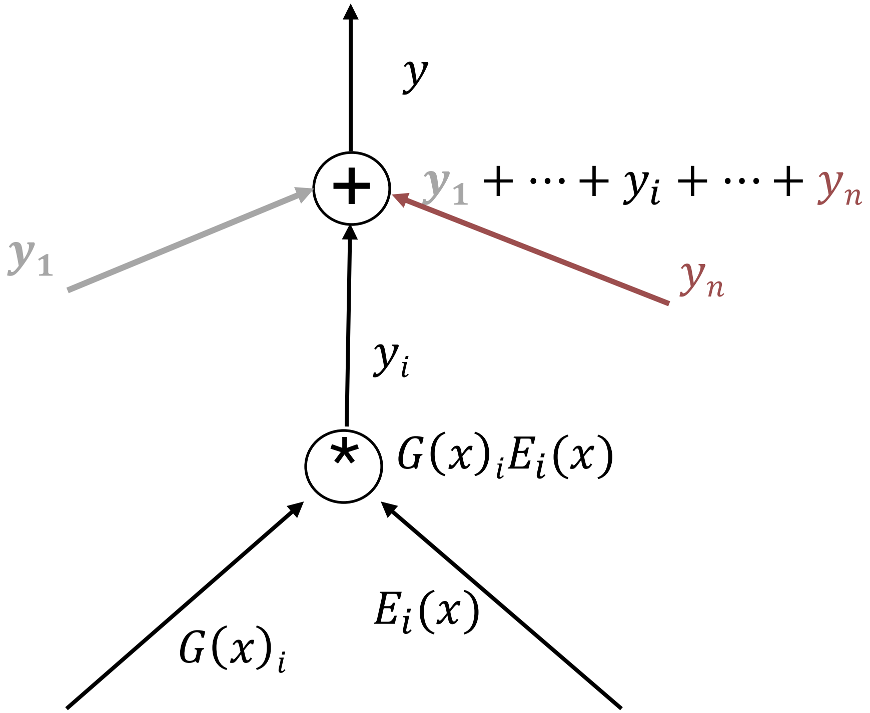

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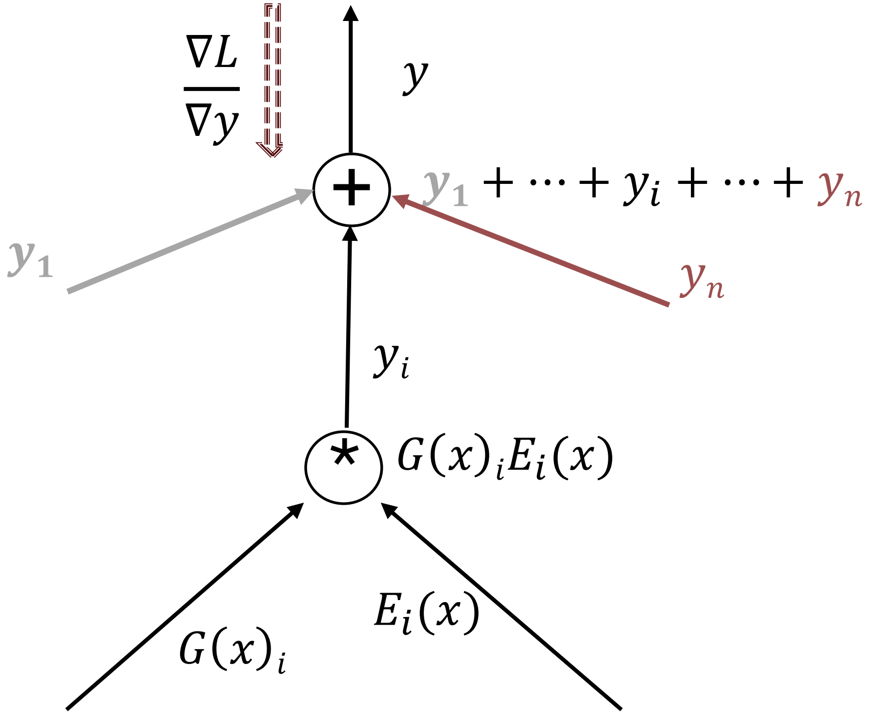

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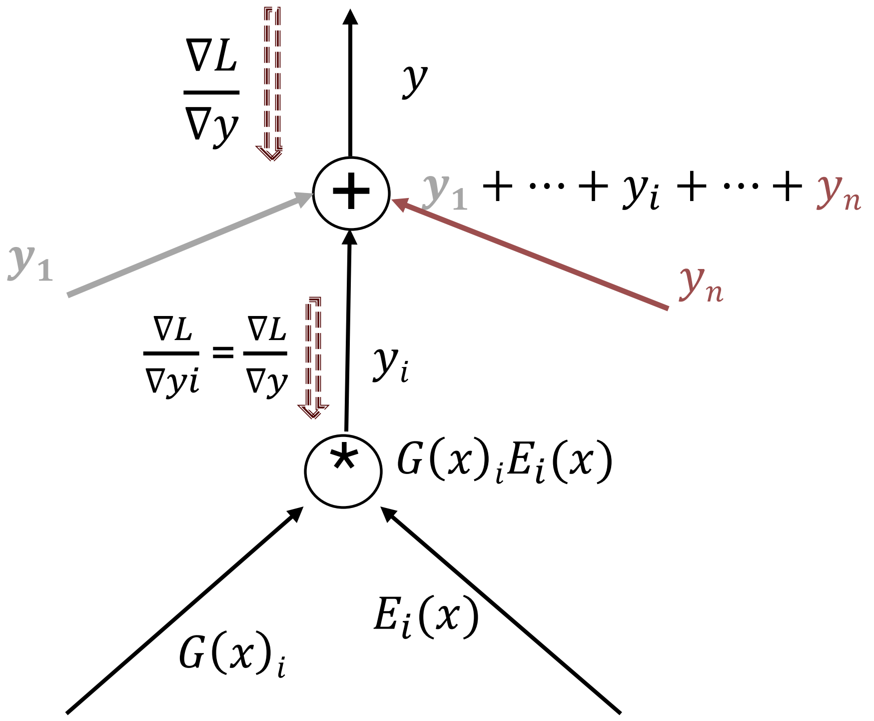

MoE Layer Backpropagation

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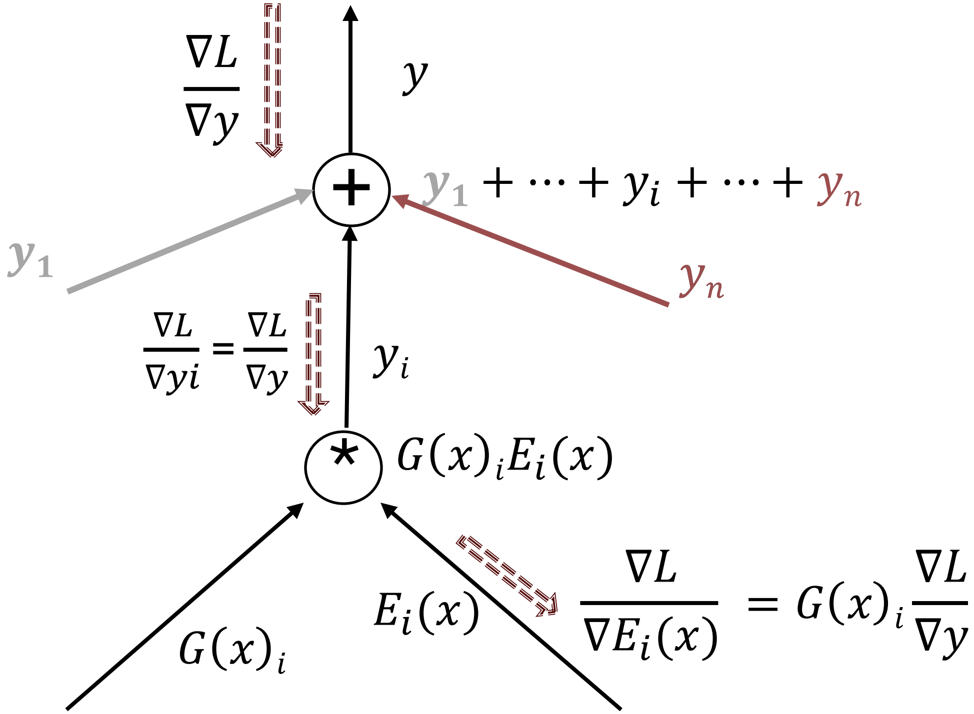
$$y = \sum_{i=1}^{n} \underbrace{G(x)_i E_i(x)}_{y_i}$$

$$\frac{\nabla L}{\nabla E_i(x)} = G(x)_i \frac{\nabla L}{\nabla y}$$

d-dim.

Scalar

d-dim. vector



MoE Layer Backpropagation

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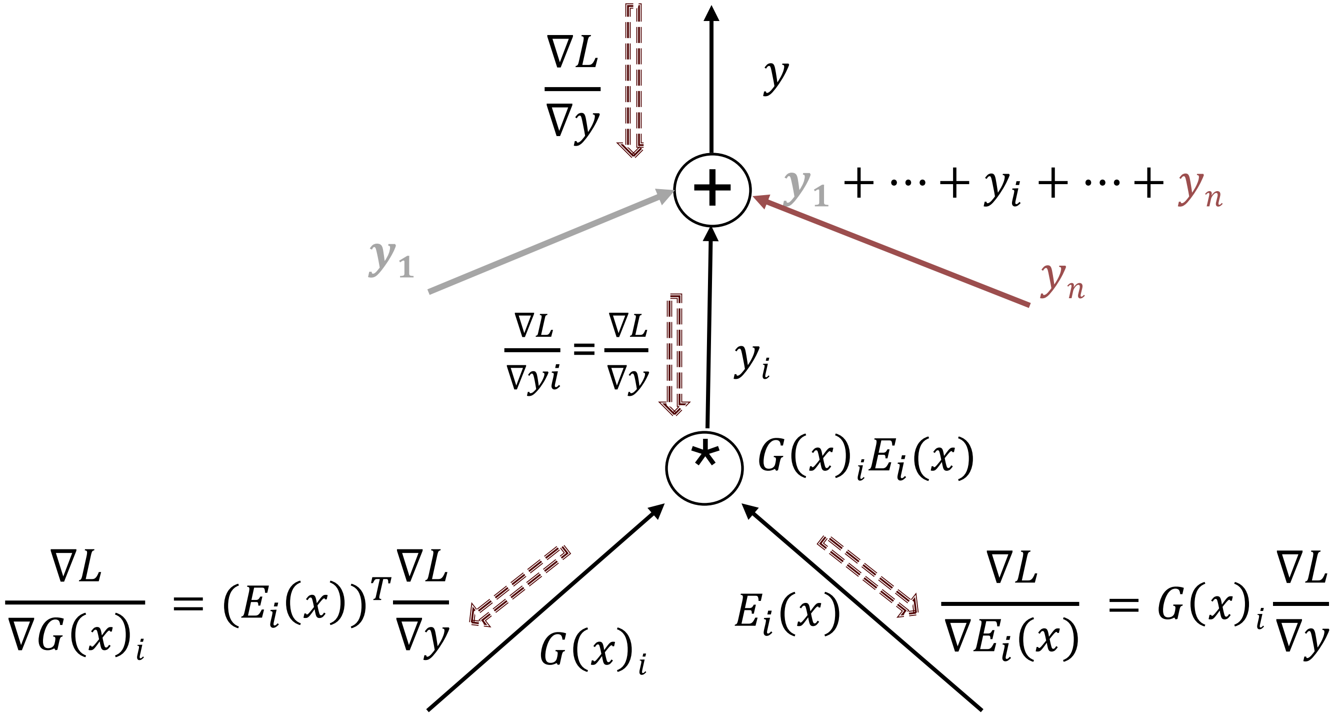
$$y = \sum_{i=1}^{n} \underbrace{G(x)_i E_i(x)}_{y_i}$$

$$\frac{\nabla L}{\nabla E_i(x)} = G(x)_i \frac{\nabla L}{\nabla y}$$

d-dim. | Scalar | d-dim. vector

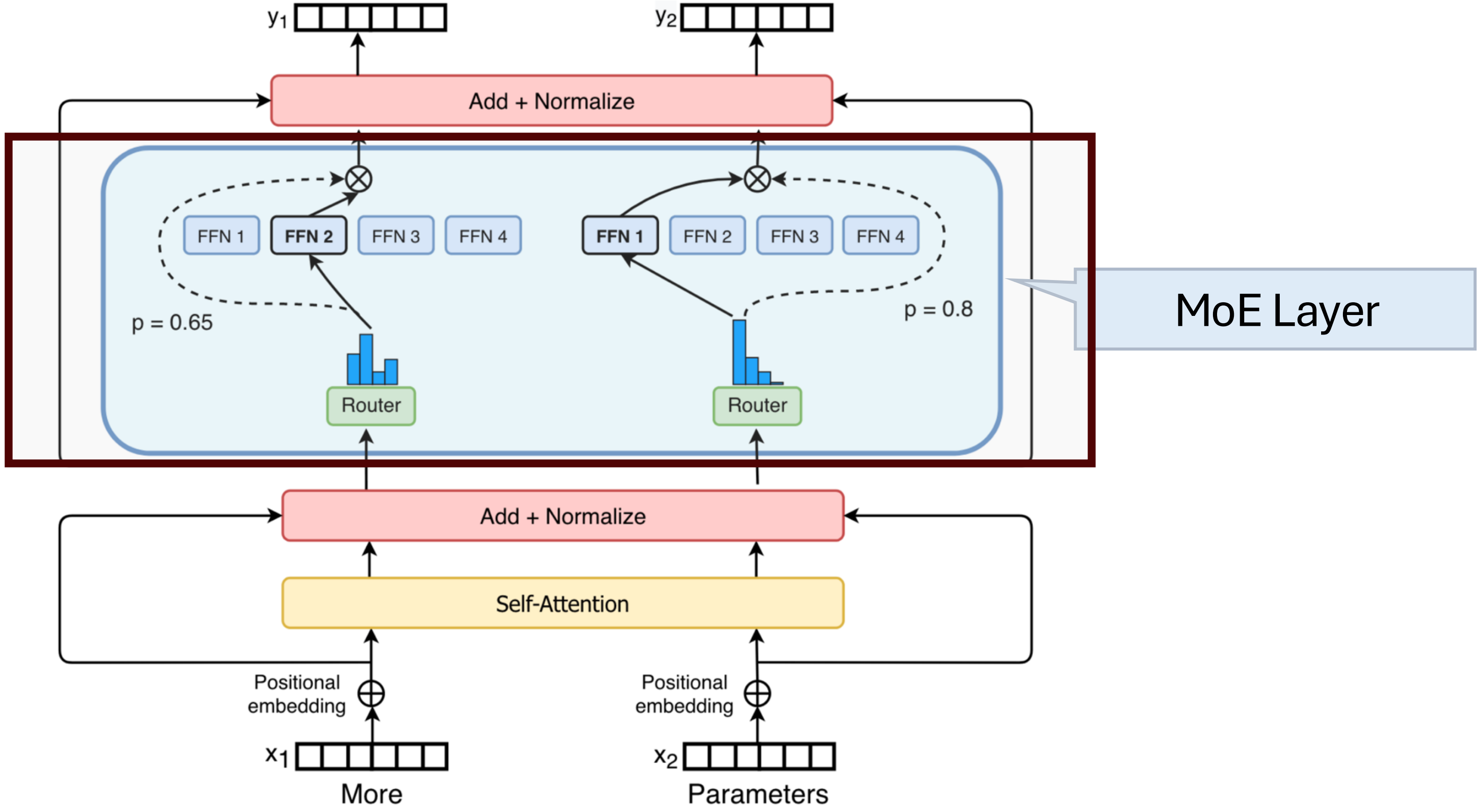
$$\frac{\nabla L}{\nabla G(x)_i} = (E_i(x))^T \frac{\nabla L}{\nabla y}$$

Scalar | d-dim. vector | d-dim. vector



Sparse MoE Layer – Greedy expert selection

$$G_\sigma(x) = \text{Softmax}(x \cdot W_g)$$



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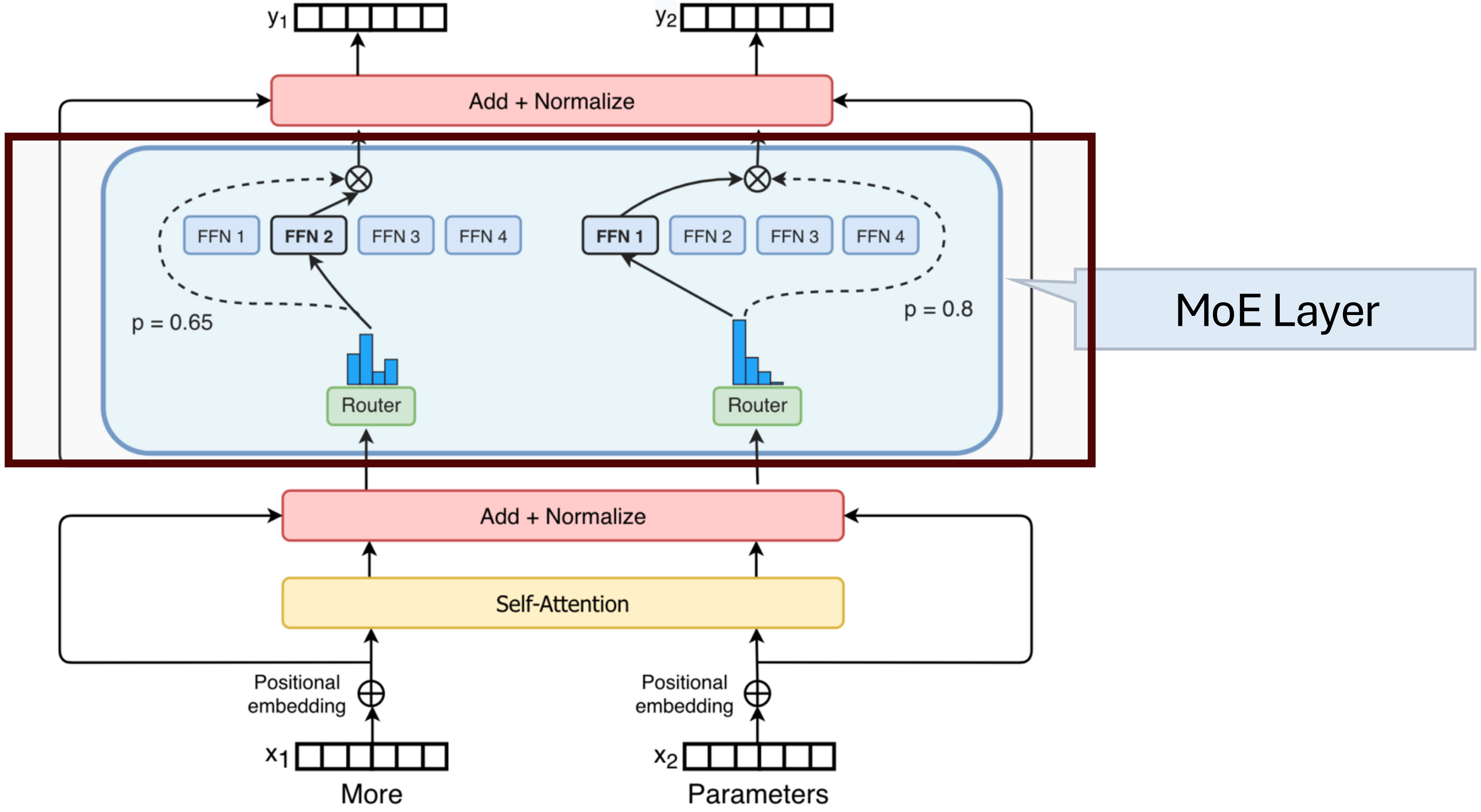
$$i* = argmax\ G_\sigma(x)$$



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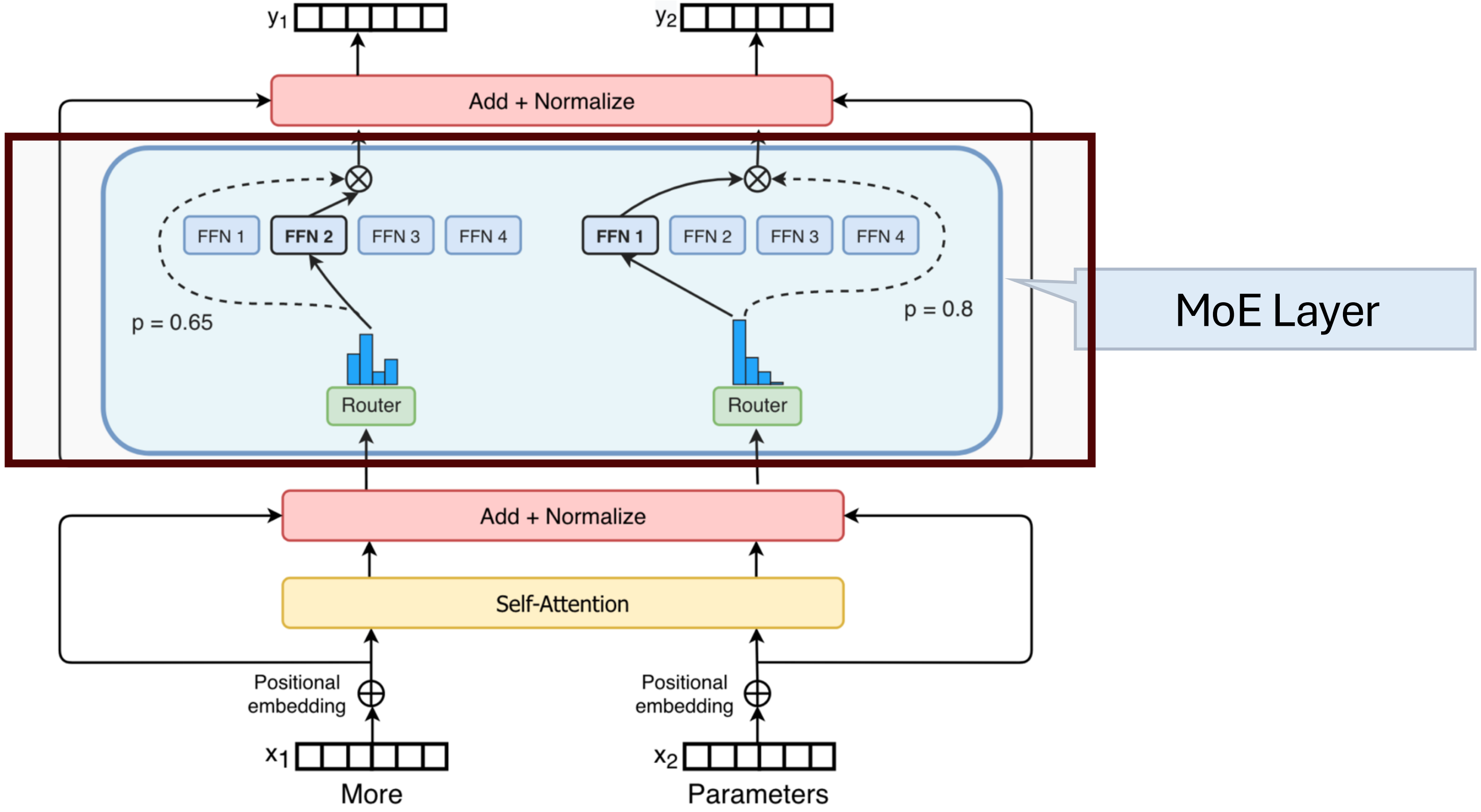
$$i* = argmax\ G_\sigma(x)$$

$$y = G(x)_{i*}E_{i*}(x)$$



Sparse MoE Layer – Greedy expert selection

$$G_\sigma(x) = \mathrm{Softmax}(x \cdot W_g)$$

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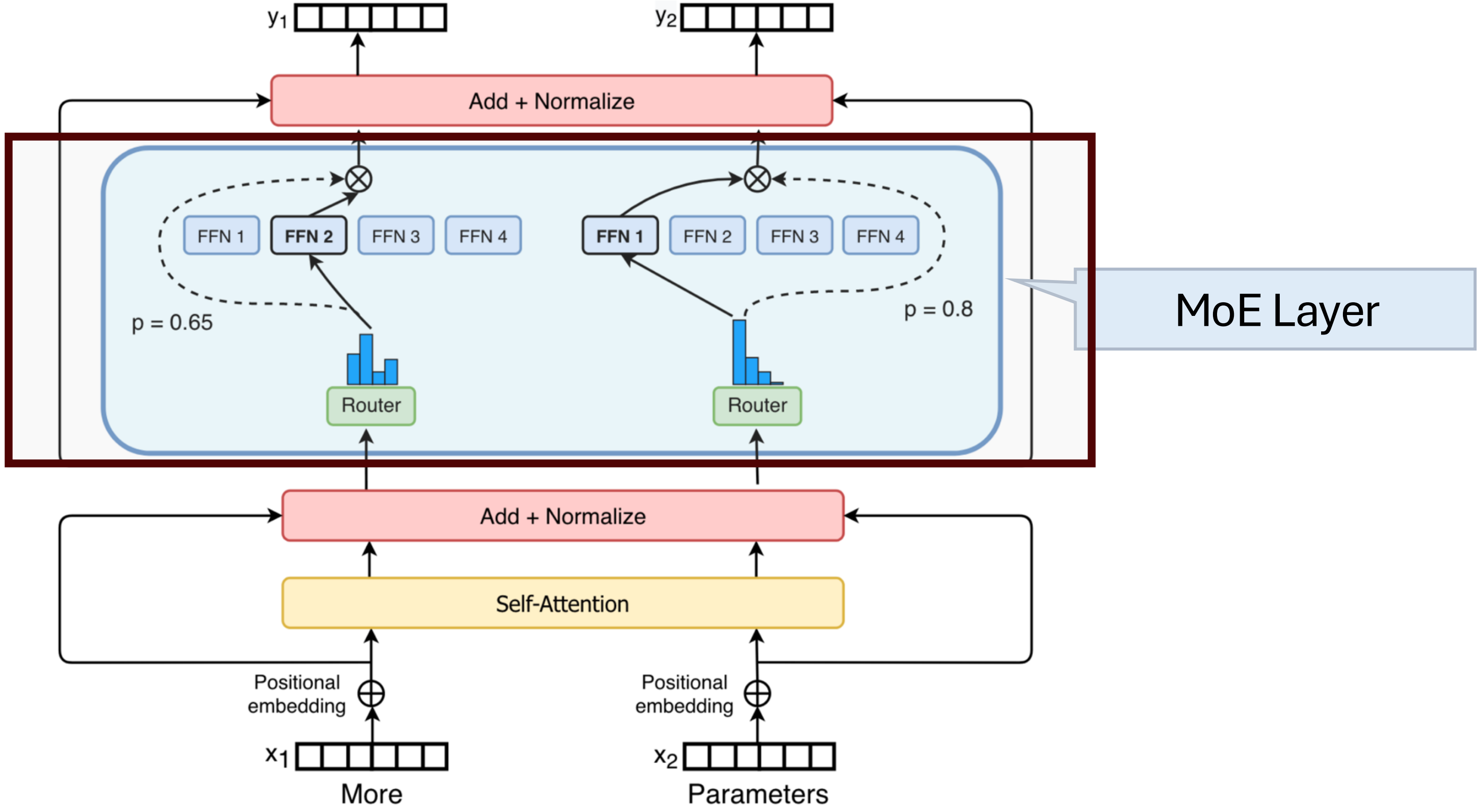
$$y = G(x)_{i*} E_{i*}(x)$$

$$\frac{\nabla L}{\nabla E_{i*}(x)} = G(x)_{i*} \frac{\nabla L}{\nabla y}$$



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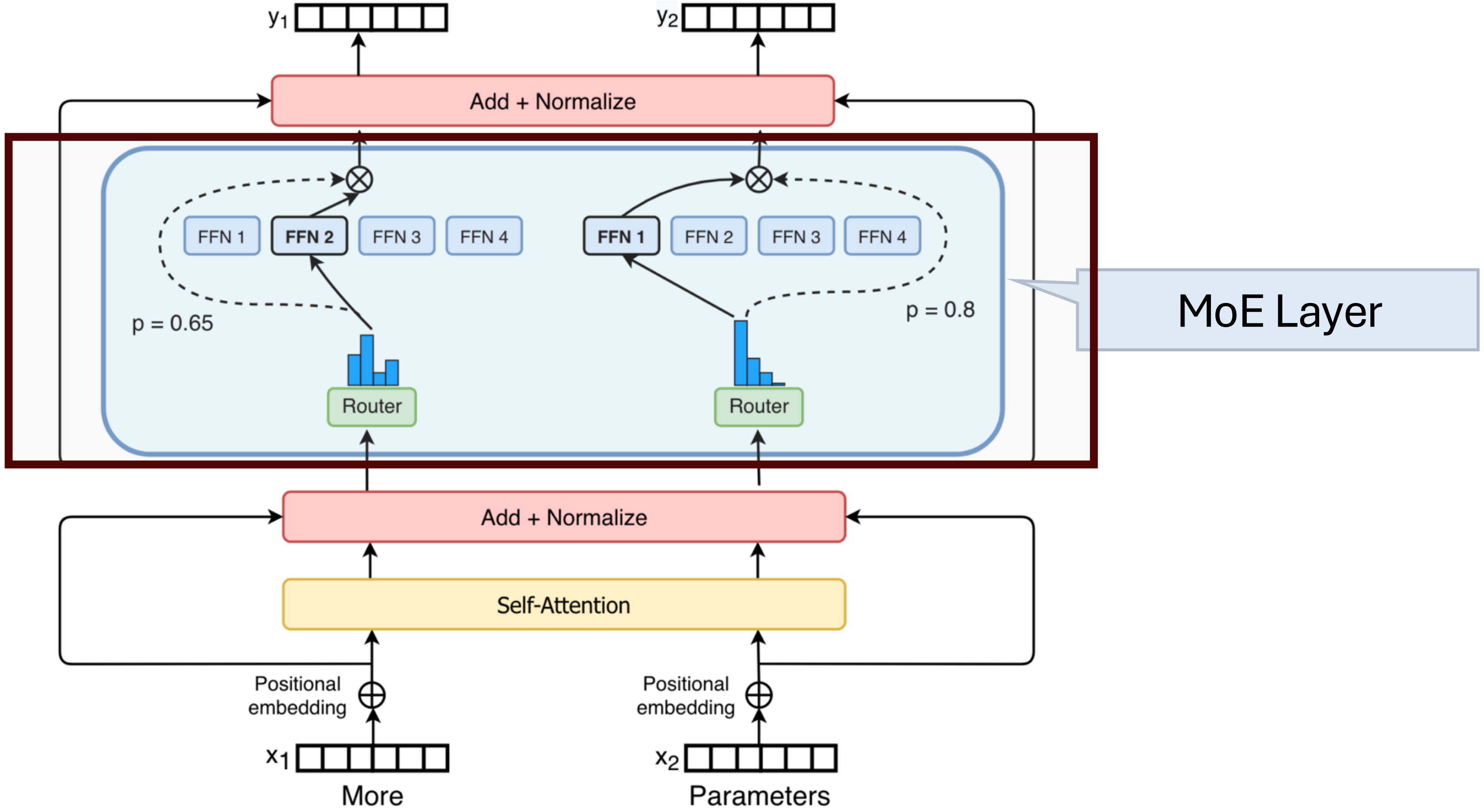

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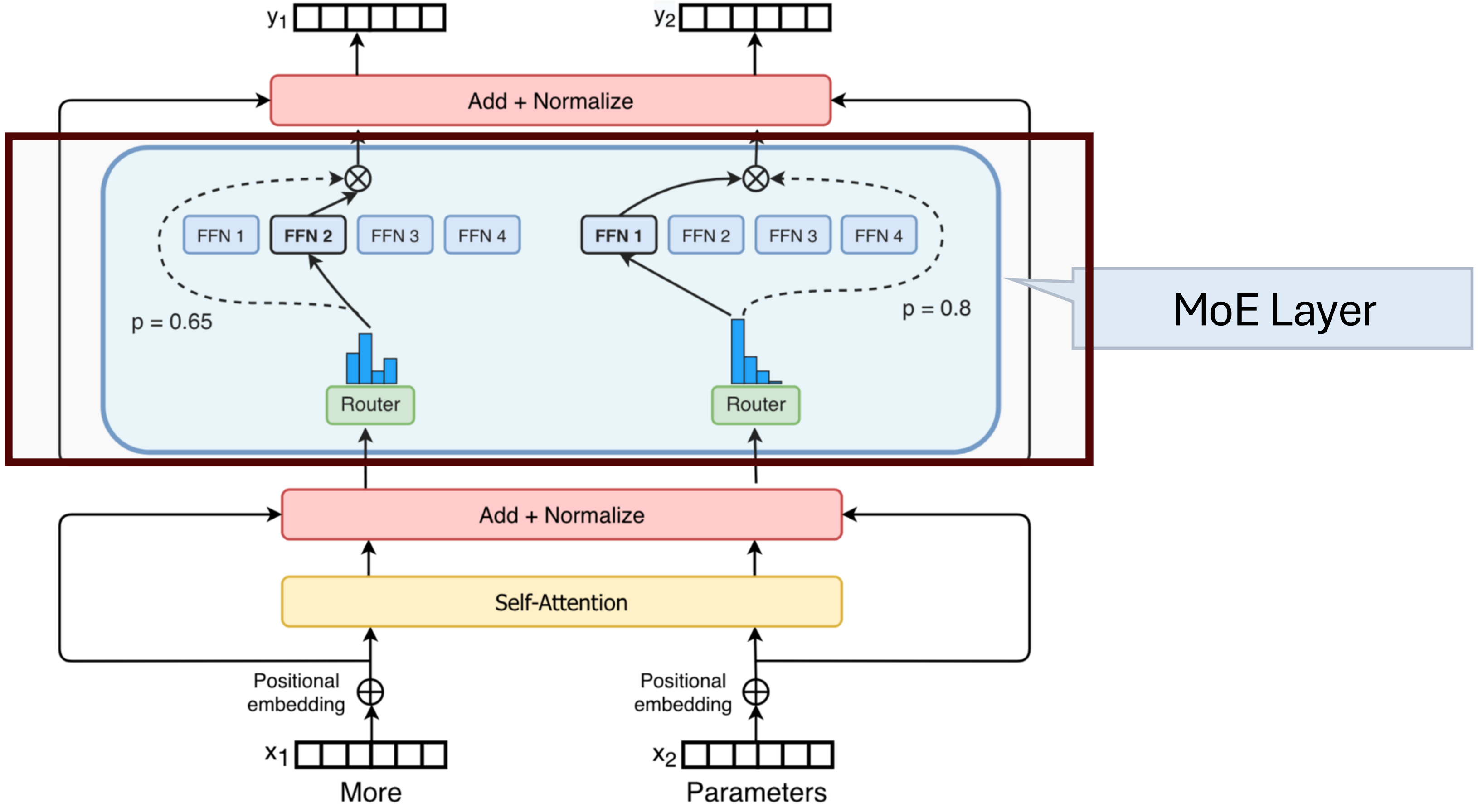
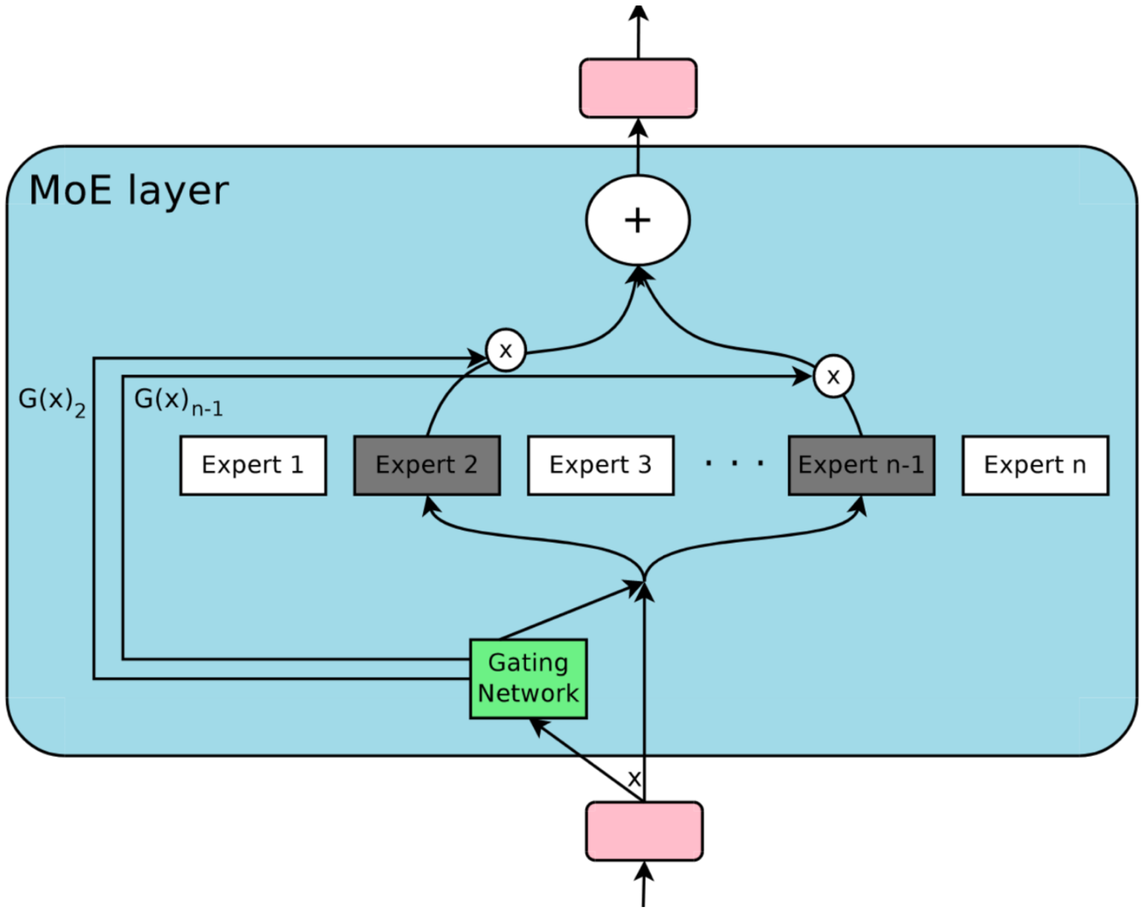
$$\frac{\nabla L}{\nabla E_i(x)} = \mathbf{0}\,;\ \frac{\nabla L}{\nabla G(x)_i} = 0\ for\ i\ \neq i*$$



Sparse MoE Layer - Noisy Top-K Gating



Content credits: Outrageously Large Neural Networks: The Sparsely-Gated Mixture-of-Experts Layer

Sparse MoE Layer - Noisy Top-K Gating

$$H(x)_i = (x \cdot W_g)_i$$

i^{th} logit of the Gating Network

$$G_\sigma(x) = Softmax(H(x))$$

Sparse MoE Layer - Noisy Top-K Gating

$$H(x)_i = (x \cdot W_g)_i + StandardNormal()$$

$$G_\sigma(x) = Softmax(H(x))$$

Sparse MoE Layer - Noisy Top-K Gating

$$H(x)_i = (x \cdot W_g)_i + StandardNormal() \cdot Softplus((x \cdot W_{noise})_i)$$

$$G_\sigma(x) = Softmax(H(x))$$

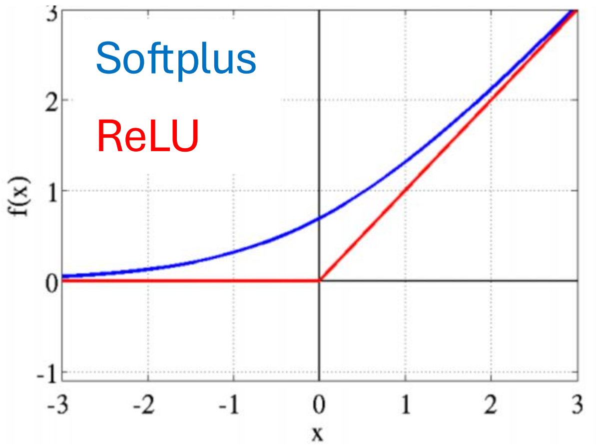
Sparse MoE Layer - Noisy Top-K Gating

$$H(x)_i = (x \cdot W_g)_i + StandardNormal() \cdot Softplus((x \cdot W_{noise})_i)$$

Learnable parameter
Mean

Learnable parameter
Std. Dev

$$\text{Softplus}(x) = \frac{1}{\beta} * \log(1 + \exp(\beta * x))$$



Sparse MoE Layer - Noisy Top-K Gating

$$H(x)_i = (x \cdot W_g)_i + StandardNormal() \cdot \boxed{Softplus}((x \cdot W_{noise})_i)$$

Learnable parameter
Mean

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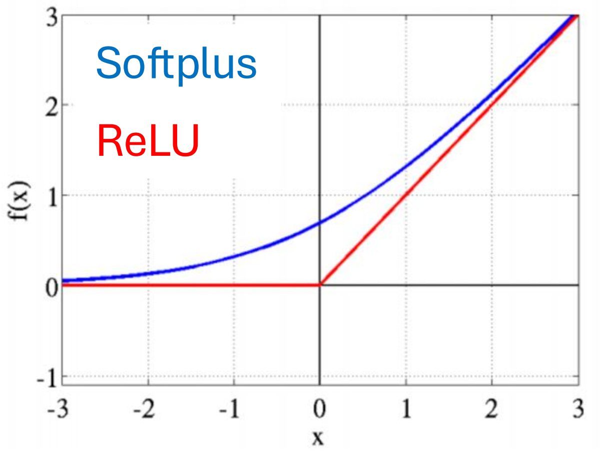
Std. Dev

Sparse MoE Layer - Noisy Top-K Gating



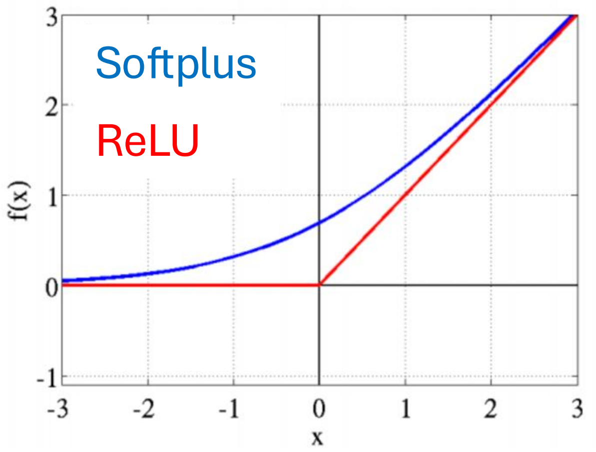
$$H(x)_i = (x \cdot W_g)_i + StandardNormal() \cdot Softplus((x \cdot W_{noise})_i)$$

Learnable parameter
Mean

Learnable parameter
Std. Dev

$$KeepTopK(v,k)_i = \begin{cases} v_i & \text{if } v_i \text{ is in the top } k \text{ elements of } v. \\ -\infty & \text{otherwise.} \end{cases}$$

Sparse MoE Layer - Noisy Top-K Gating



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Ensures 0 probability after softmax

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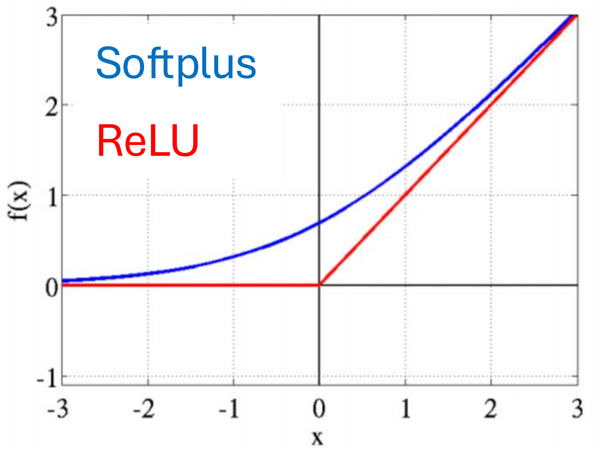

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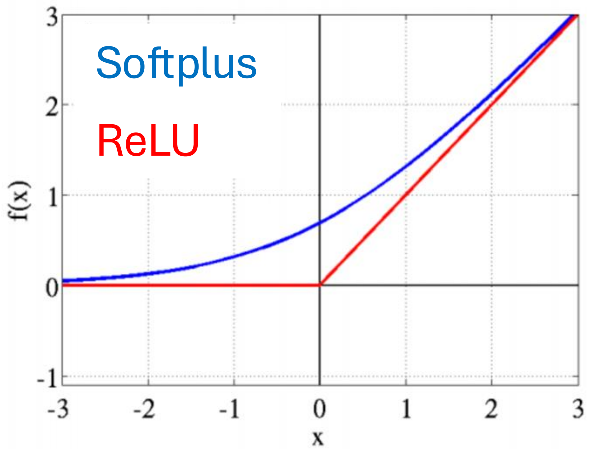
$$H(x)$$

Sparse MoE Layer - Noisy Top-K Gating



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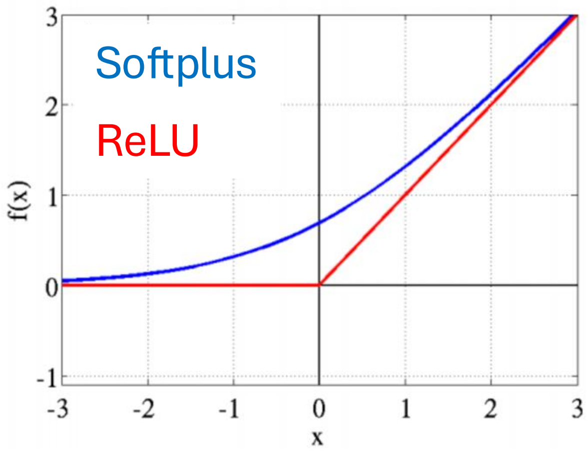
$$KeepTopK(H(x), k)$$

Sparse MoE Layer - Noisy Top-K Gating



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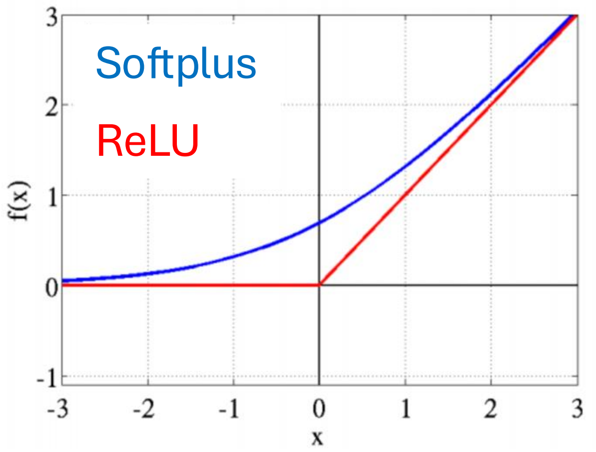

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Ensures 0 probability after softmax

$$G(x) = Softmax(KeepTopK(H(x), k))$$

Only *k* non-zero elements; add up to *1*

Why do experts not collapse in practice?

❖ All experts have:

- Same architecture
- Same initialization scheme
- Trained with same optimizer

❖ So why don't they collapse? I.e.

Expert 1 = Expert 2 = ... = Expert n -1 = Expert n ?

Why do experts not collapse in practice?

Towards Understanding the Mixture-of-Experts Layer in Deep Learning

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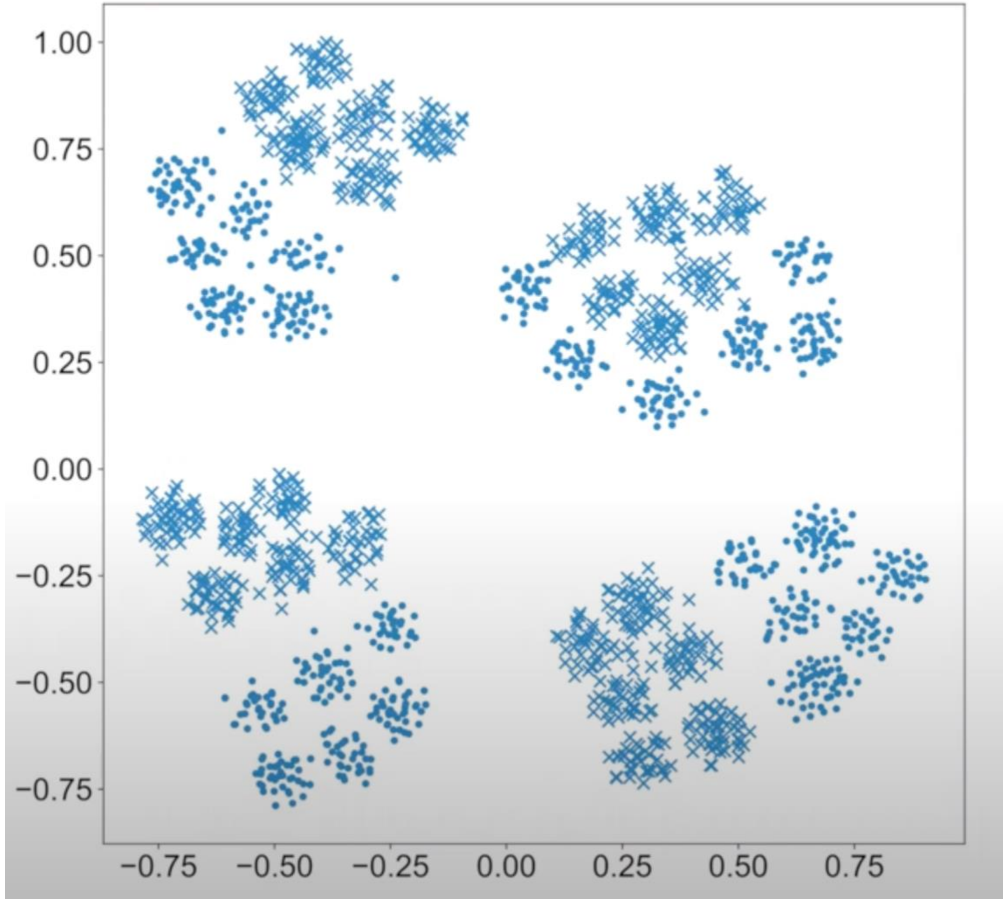
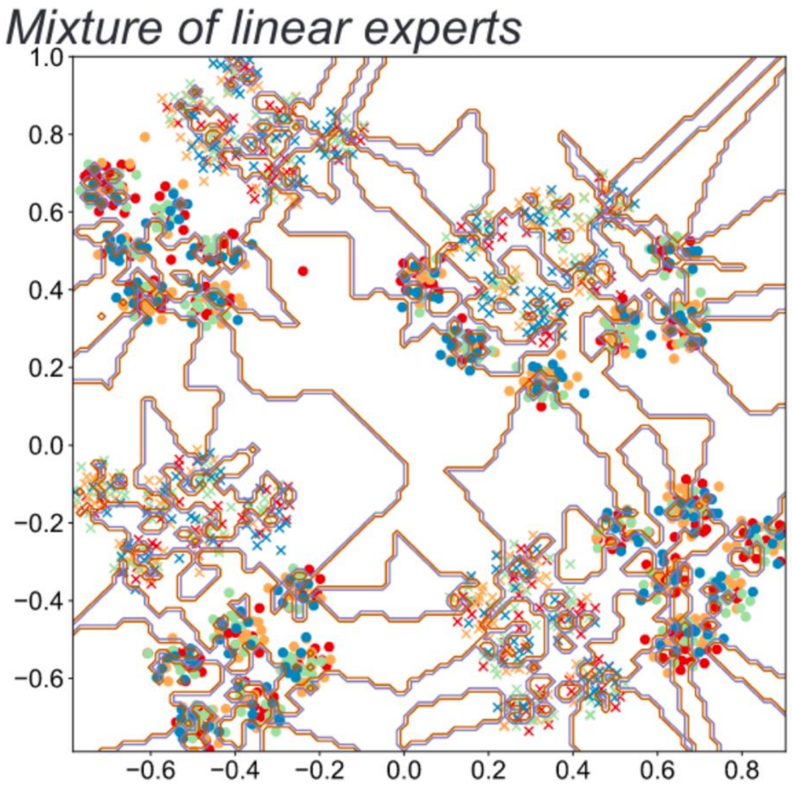
Why do experts not collapse in practice?



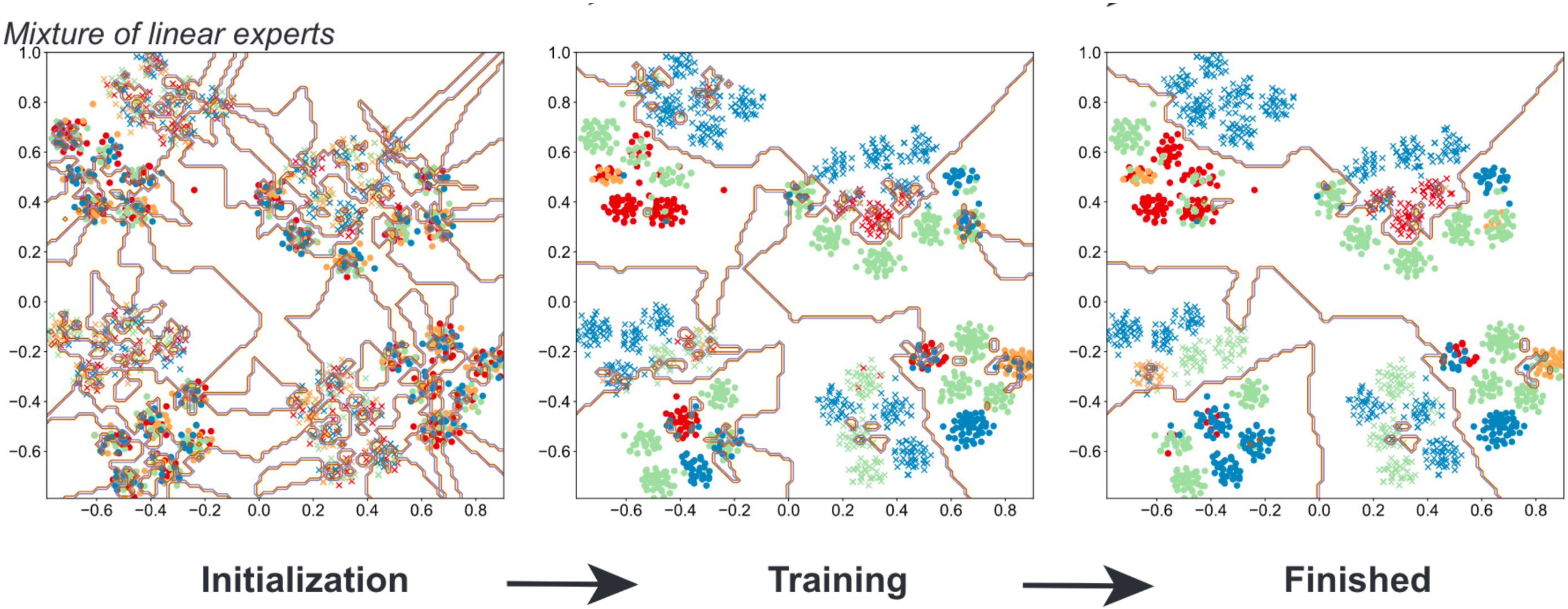
- ✓ Synthetically generated 50 dim. data
- ✓ Visualization in 2-D space
- ✓ 4 clusters
- ✓ Each cluster is linearly separable
- ✓ Ideally, a mixture of 4 linear experts sufficient for classification

Why do experts not collapse in practice?

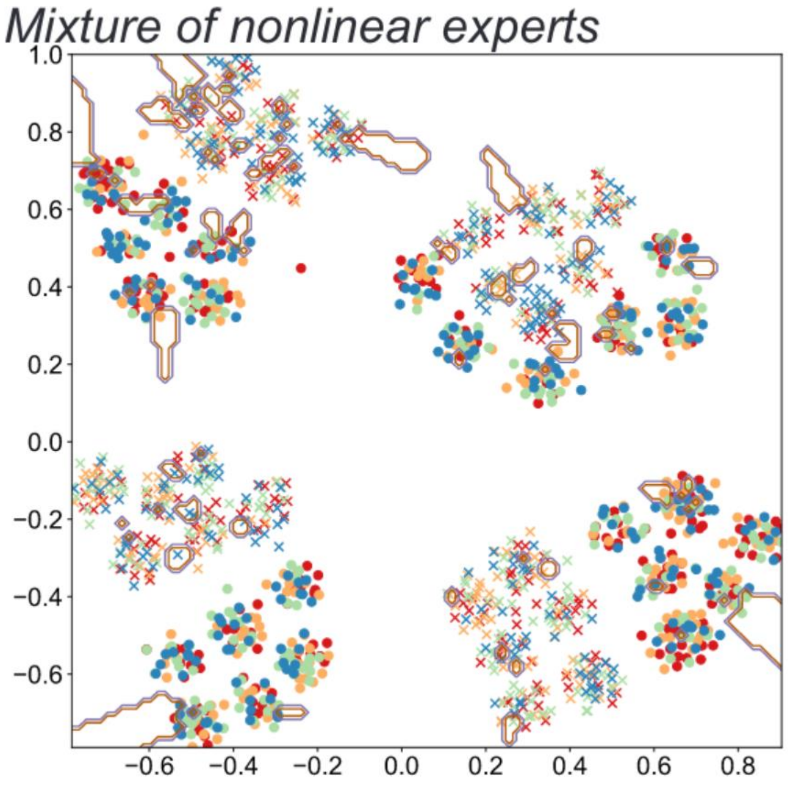


Initialization

Why do experts not collapse in practice?



Why do experts not collapse in practice?



Initialization

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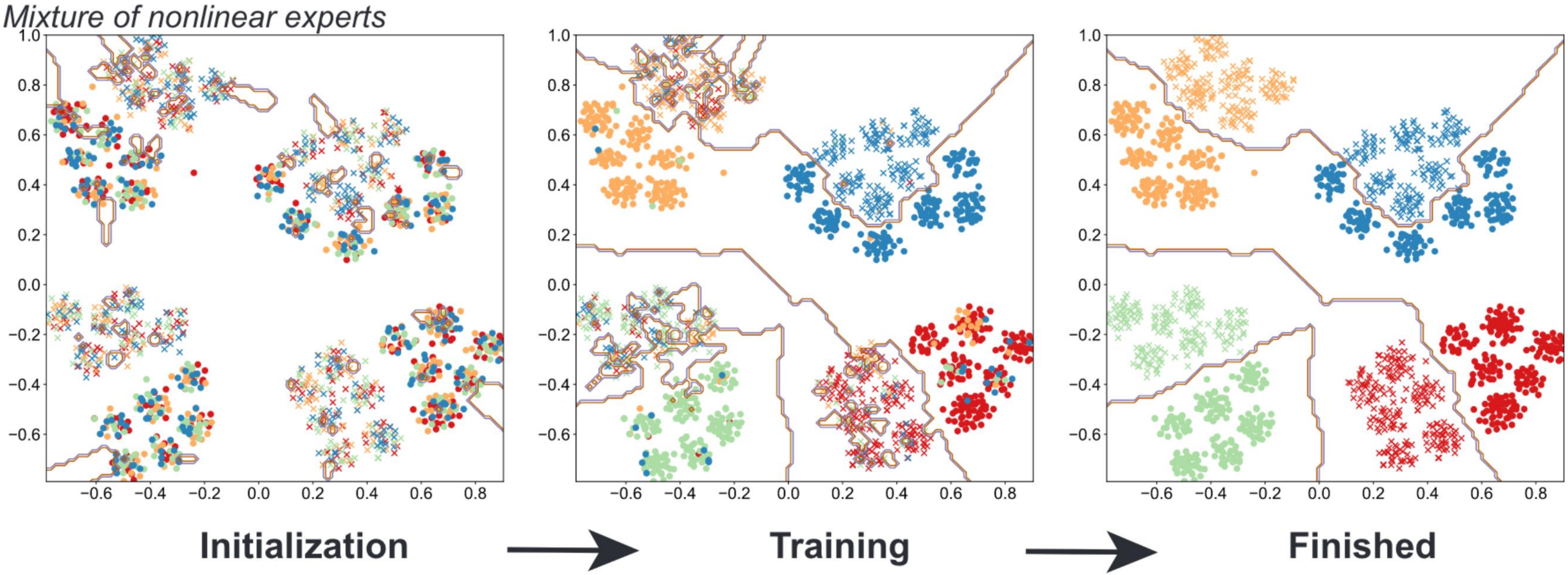
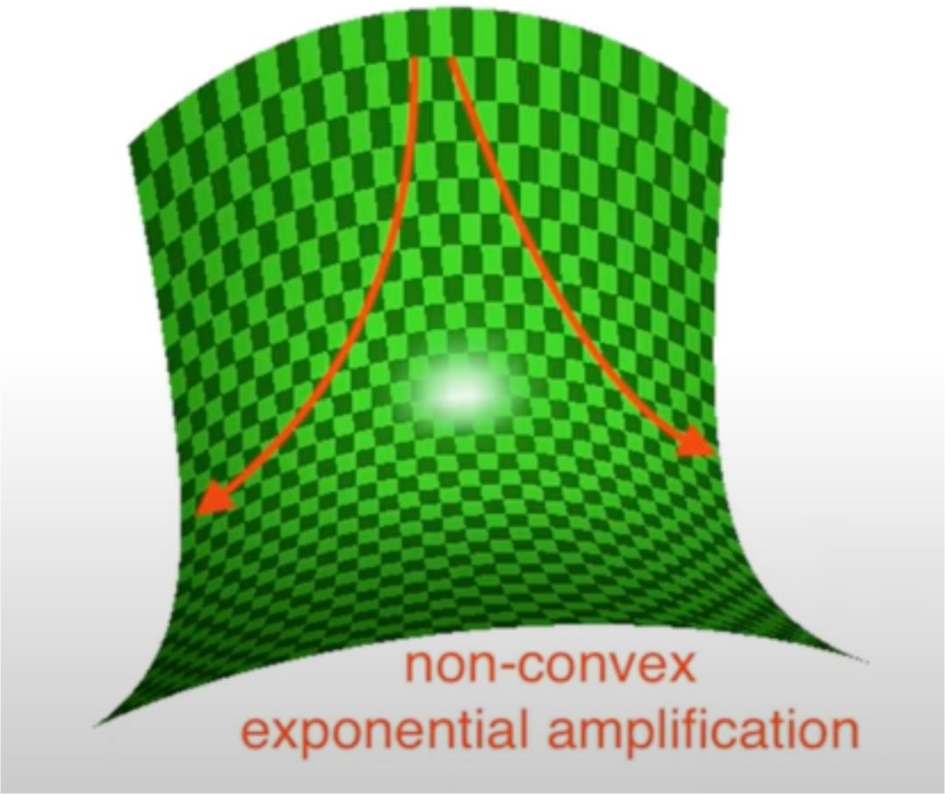
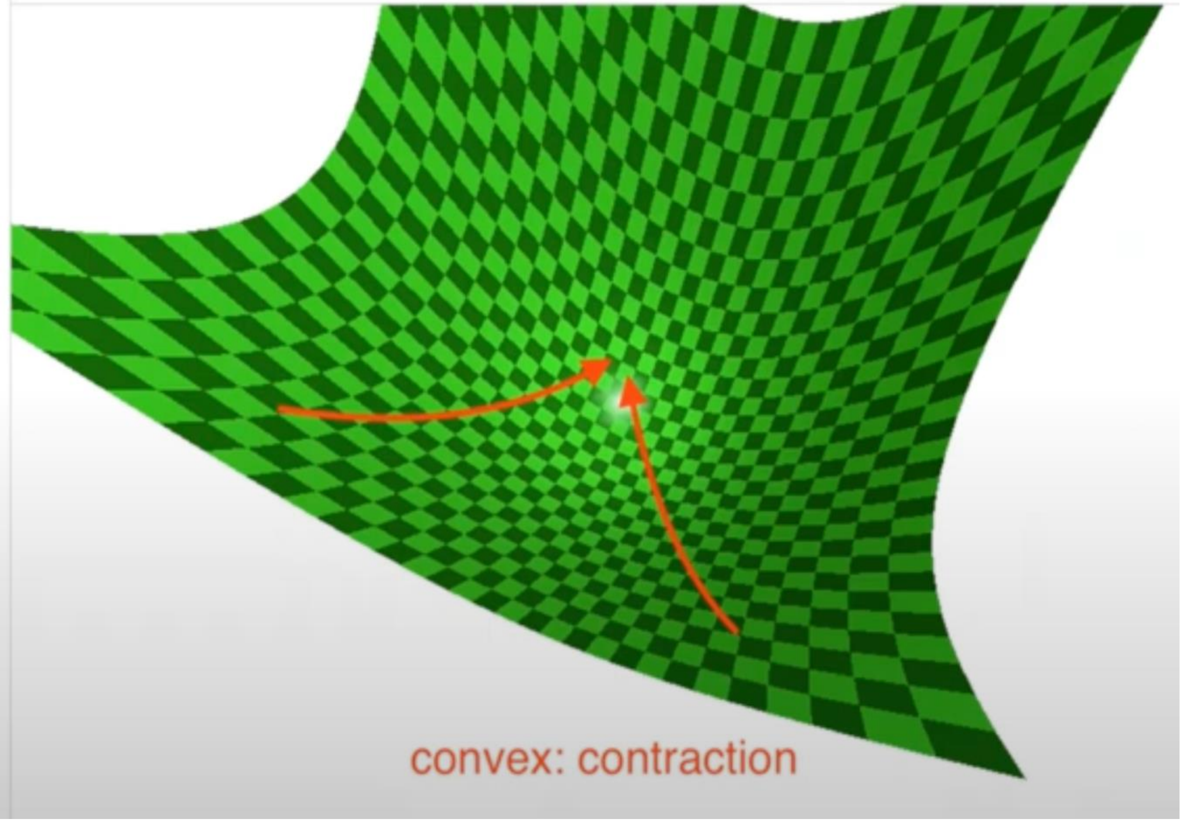


Why do experts not collapse in practice?





Learning dynamics of experts and router

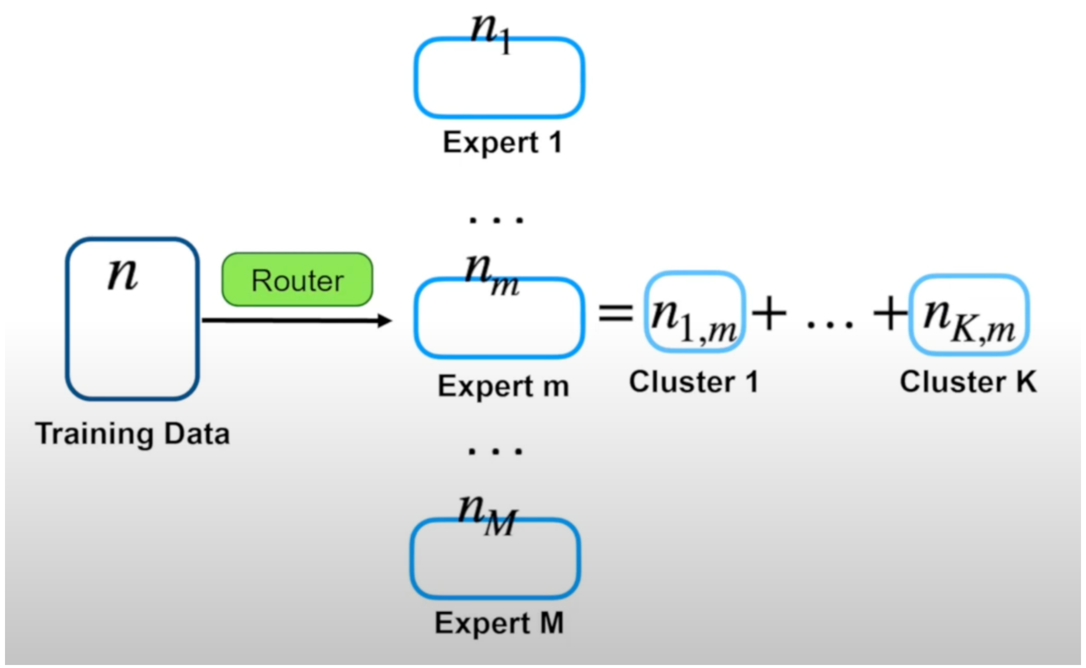
- Exploration stage:
  - experts diverge; router nearly untrained
- Router learning stage:
  - router learns to dispatch

Learning dynamics of experts and router

- **Exploration stage:**
  - experts diverge; router nearly untrained

- **Router learning stage:**
  - router learns to dispatch

$n_{k,m}$: # of samples from Cluster k routed to Expert m
n_m : # of samples routed to Expert m $= \sum_{k=1}^{K} n_{k,m}$
n : # of total samples $= \sum_{m=1}^{M} n_m$



Learning dynamics of experts and router

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- Router learning stage:
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$$\text{entropy} = -\sum_{m=1, n_m \neq 0}^{M} \frac{n_m}{n} \sum_{k=1}^{K} \frac{n_{k,m}}{n_m} \cdot \log\left(\frac{n_{k,m}}{n_m}\right)$$

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Learning dynamics of experts and router

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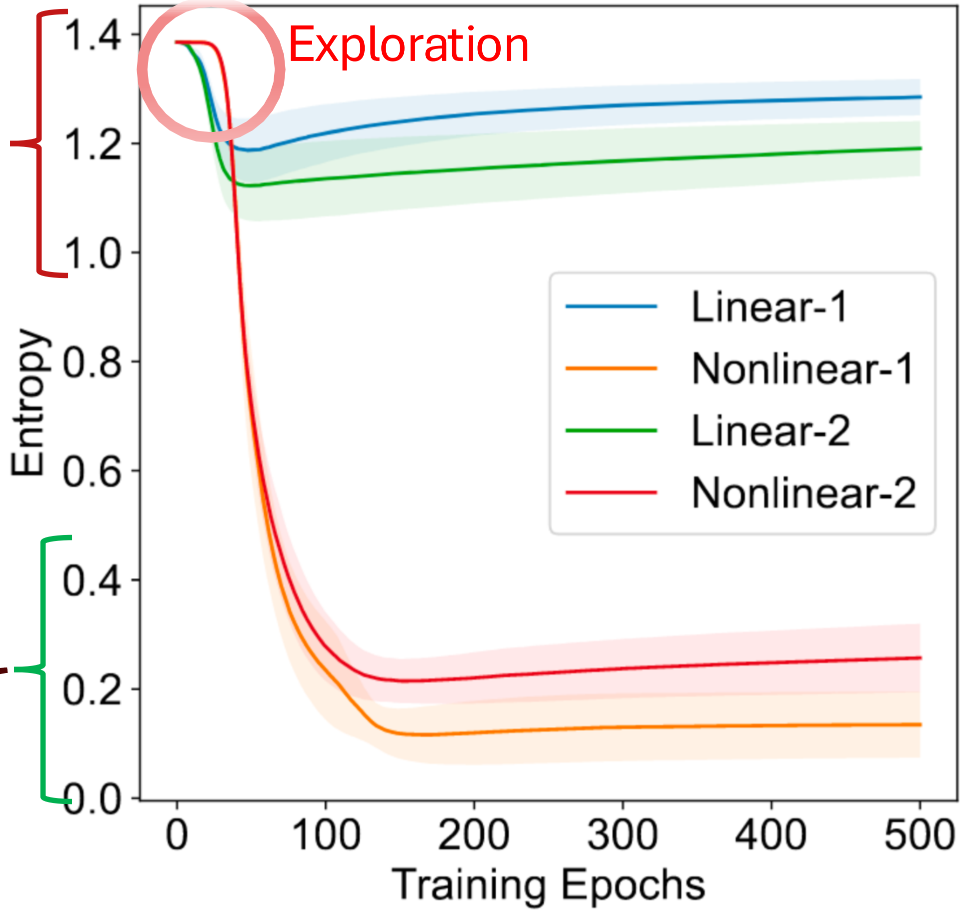
Entropy is high if an input from cluster *k* is routed uniformly to all the experts

Entropy is low if an input from cluster k is routed to one expert



Pros and Cons of Sparse MoE Layer

Pros

👍 Increased model parameters

👍 Efficient pretraining due to conditional (sparse) computation

👍 Faster inference

Cons

👎 Unstable training

- ☹ Router collapse– router sends all tokens to the same expert
- ☹ May diverge

👎 High memory requirement - all parameters need to be loaded in vRAM (GPU memory)