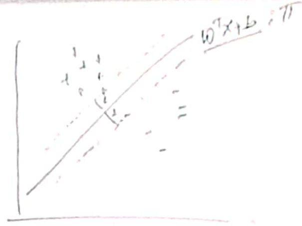
— Support Vector Machines —

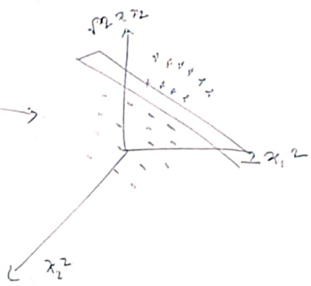
$$= \sum_{i=1}^{m} \alpha_i - \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{m} \alpha_i \alpha_j y^i y^j \langle x_i , x_j \rangle$$

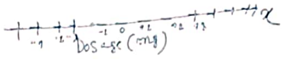
Kernel

w^*, α_i^*, b^*

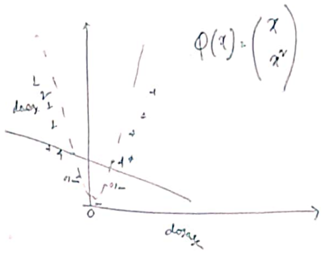
$$\varphi\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1^2 \\ \sqrt{2}\, x_1 x_2 \\ x_2^2 \end{pmatrix}$$







$$\varphi(x) = \begin{pmatrix} x \\ x^2 \end{pmatrix}$$



$$\min \frac{1}{2} \|w\|^2$$

$$\text{s.t. } y^i (w^T x_i + b) \geq 1$$

$g_i(w)$

KKT Condn $\alpha_i \, g_i(w) = 0$

1) $g_i(w) = 0$
$\alpha_i > 0$

2) $\alpha_i = 0$

	x_1	x_2
a:	a_1 1	5 a_2
b:	2 b_1	6 b_2

$$\varphi(a) = \begin{pmatrix} a_1^2 \\ \sqrt{2}\, a_1 a_2 \\ a_2^2 \end{pmatrix}$$

$$\varphi(b) = \begin{pmatrix} b_1^2 \\ \sqrt{2}\, b_1 b_2 \\ b_2^2 \end{pmatrix}$$

$$\varphi(a) \cdot \varphi(b)$$

$$= a_1^2 b_1^2 + 2 a_1 a_2 b_1 b_2 + a_2^2 b_2^2$$

$$= (a_1 b_1 + a_2 b_2)^2$$

$$= \left(\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \right)^2$$

$$K(a,b) = \underline{(\vec{a} \cdot \vec{b})^2}$$

is a function that takes input as a vector in the original space and returns the dot product of vectors in the high dimensional space

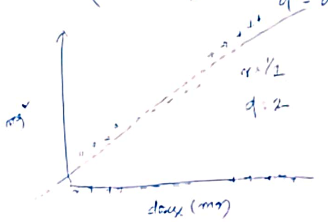
Polynomial $(a \cdot b + r)^d$ r: coeff d: deg

linear $a \cdot b$

RBF: $\exp(-\gamma \|a-b\|^2)$

— Support Vector Machines —

$(a \times b + r)^d$ $\quad$ r = coeff of the polynomial.
d = deg of the polynomial



$r = 1$

$d = 2$

$$(a \times b + 1)^2$$
$$= 2ab + a^2b^2 + 1$$
$$= \begin{pmatrix} \sqrt{2}a \\ a^2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} \sqrt{2}b \\ b^2 \\ 1 \end{pmatrix}$$
$$= \begin{pmatrix} \sqrt{2}a \\ a^2 \end{pmatrix} \begin{pmatrix} \sqrt{2}b \\ b^2 \end{pmatrix}$$

$$\left(a \times b + \tfrac{1}{2}\right)^2 = \left(a \times b + \tfrac{1}{2}\right)\left(a \times b + \tfrac{1}{2}\right)$$
$$= a^2b^2 + \tfrac{1}{2}ab + \tfrac{1}{2}ab + \tfrac{1}{4}$$
$$= a^2b^2 + ab + \tfrac{1}{4}$$
$$= ab + a^2b^2 + \tfrac{1}{4}$$
$$= \begin{pmatrix} a \\ a^2 \\ 1/2 \end{pmatrix} \cdot \begin{pmatrix} b \\ b^2 \\ 1/2 \end{pmatrix}$$
$$= \begin{pmatrix} a \\ a^2 \end{pmatrix} \begin{pmatrix} b \\ b^2 \end{pmatrix}$$

Polynomial: $(\gamma\, a\cdot b + r)^d$ $\quad r = \text{coeff} \quad d = \text{deg}$

Linear: $a\cdot b$

RBF: $\exp\left(-\gamma \|a-b\|^2\right)$

Gaussian: $\exp\left(-\dfrac{\|a-b\|^2}{2\sigma^2}\right)$

Sigmoid: $\tanh\left(\gamma\, a\cdot b + c\right)$

→ $e^{-\gamma(a-b)^2}$ γ = scalar value, which depicts the influence of a.b

* It transforms datapoints to <u>infinite dim</u> space

* locally weighted Neighbourhood neighbour.

$a = 2.5$ $\gamma = 1$: $K = 0.11$ ✓
$b = 4$ $\gamma = 2$: $K = 0.01$

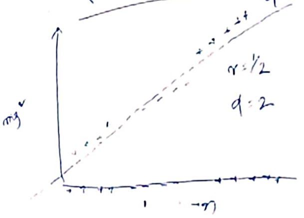
$a = 2.5$ $K \Rightarrow 0$
$b = 16$
$\gamma = 1$

– Support Vector Machines –

$(a \cdot b + r)^d$

r = coeff of the polynomial

d = deg of the polynomial



$r = 0$	$a^d b^d$
$d = 1$	$a\,b$
$d = 2$	$a^2 b^2$
$d = 3$	$a^3 b^3$
$d = \infty$	$a^\infty b^\infty$

$$\boxed{\begin{aligned} &ab + a^2 b^2 + a^3 b^3 + a^4 b^4 + \dots + a^\infty b^\infty \\ &= (a, a^2, a^3, \dots, a^\infty) \cdot (b, b^2, b^3, \dots, b^\infty) \end{aligned}}$$

$$e^{-\gamma (a-b)^2} \qquad \gamma = 1/2$$

$$= e^{-\gamma (a^2 + b^2 - 2ab)} = e^{-\frac{1}{2}(a^2 + b^2)}\, e^{ab}$$

Taylor Series

$$f(x) = f(a) + \frac{f'(a)(x-a)}{1!} + \frac{f''(a)(x-a)^2}{2!} + \dots + \frac{f^{\infty}(a)}{\alpha!}(x-a)^\infty$$

$f(a)$ = exists

$$e^x = e^a + \frac{e^a}{1!}(x-a) + \frac{e^a}{2!}(x-a)^2 + \dots$$

$a = 0$

$e^0 = 1$

$$= 1 + \frac{1}{1!}x + \frac{1}{2!}x^2 + \dots$$

$$e^{ab} = 1 + \frac{1}{1!}ab + \frac{1}{2!}(ab)^2 + \dots + \frac{1}{d!}(ab)^\infty$$

$\rightarrow e^{-\gamma(a-b)^2}$ γ = scalar value which depotes the influence of $a \cdot b$

* It transforms datapoints to infinite dim space

* locally weighted Neighest neighbour

$a = 2.5$ $\gamma = 1 : K = 0.11$

$b = 4$ $\gamma = 2 : K = 0.01$

$a = 2.5$ $K \rightarrow 0$

$b = 16$

$\gamma = 1$

$$\gamma = \sqrt{e^{-\frac{1}{2}(a^2+b^2)}}$$

$$\left(1, \sqrt{\frac{1}{1!}}\, a, \sqrt{\frac{1}{2!}}\, a^2, \ldots, \sqrt{\frac{1}{\infty!}}\, a^{\infty}\right)$$

$$\left(1, \frac{1}{\sqrt{1!}}\, b, \sqrt{\frac{1}{2!}}\, b^2, \ldots, \sqrt{\frac{1}{\infty!}}\, b^{\infty}\right)$$

Polynomial: $(\gamma\, a \cdot b + r)^d$ r = coeff d = deg

linear: $a \cdot b$

RBF: $\exp\left(-\gamma \|a-b\|^2\right)$

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