optimization with inequality Constraints.

1.1 Convert it to Lagrange ~~Mat~~ Eq. (L)

1.2 Derivative of L w.r.t all variables in the objective fn.

2 $\lambda_i h_i = 0$

3 $h_i \le 0$

4. $\lambda_i \geq 0$

$f(): -x_1^2 - x_2^2 - x_3^2 + 4x_1 + 6x_2$

s.t.: $x_1 + x_2 \le 2$

$2x_1 + 3x_2 \le 12$

$x_1, x_2 \geq 0$

K : Karush

K : Kuhn

T : Tucker.

Condn 1

$L(x_1, x_2, x_3, \lambda_1, \lambda_2) = -x_1^2 - x_2^2 - x_3^2 + 4x_1 + 6x_2 - \lambda_1($

$$\frac{\partial L}{\partial x_1} = -2x_1 + 4 - \lambda_1 - 2\lambda_2 = 0$$

$$\frac{\partial L}{\partial x_2} = -2x_2 + 6 - \lambda_1 - 3\lambda_2 = 0$$

$$\frac{\partial L}{\partial x_3} = -2x_3 = 0 \Rightarrow x_3 = 0$$

Cond 2 $\lambda_1(x_1+x_2-2)=0$ — (2a)

$\lambda_2(2x_1+3x_2-12)=0$ — (2b)

Cond 3 $x_1+x_2-2\le 0$ — (3a)

$2x_1+3x_2-12\le 0$ — (3b)

Cond 4 $\lambda_1\ge 0$; $\lambda_2\ge 0$

Case 1 X

$\lambda_1=0$

$\lambda_2=0$

1-a, 1-b $x_1=2$

$x_2=3$

(3a) $2+3-2\le 0$ X

Case 2

$\lambda_1\ne 0$

$\lambda_1\ne 0$

from 2a, 2b

$x_2=8,\ x_1=-6$

$\lambda_2=-26$ X

Case 3

$\tau_1 = 0$

$\tau_2 \neq 0$

1a, 1b

$x_1 = \frac{2}{3} x_2$

(2b) $x_2 = 3, x_1 = 2$

(3a) $5 - 2 \leq 0$ X

Case 4

$\tau_1 \neq 0$

$\tau_2 \neq 0$

$x_1 = 1/2$ ✓

$x_2 = 3/2$ ✓

(3a) $\frac{1}{2} + \frac{3}{2} - 2 = 0$ ✓

(3b) $-13 < 0$