Generative Learning Algorithms

- Generative Classifier/Model
- Discriminative Model

$$E(x) = \mu = \int x \, p(x) \, dx$$

Co Variance.

$$Cov(x) = E\left((x - E[x])(x - E(x))^T\right)$$

$$= E(xx^T) - E(x)(E(x))^T$$

Multivariate Normal Distribution

$$p(x, \mu, \Sigma) = \frac{1}{(2\pi)^{d/2} |\Sigma|^{1/2}} \exp\left(-\frac{1}{2}(x-\mu)^T \Sigma^{-1} (x-\mu)\right)$$

determinant of Σ.

d = dimension

Vec $|_d$

vector $|_d$

matrix $| \mathbb{R}^{d \times d}$

$$p(x | y=c, \theta) \sim \mathcal{N}(x | \mu_c, \Sigma_c)$$

$$p(y=c | x ; \theta) \propto \pi_c \, \mathcal{N}(x | \mu_c, \Sigma_c)$$

$p(y=c)$

Σ = Covariance Matrix

1: μ_1, Σ_1

2: μ_2, Σ_2

$$p(x | \mu_2, \Sigma_2) = \frac{1}{(2\pi)^{d/2} \Sigma_2^{1/2}} \exp\left(-\right.$$

$$\log(p(y_1 | x)) = \log(p(y) \, p(x|y))$$

$$= \log(p(y)) + \log p(x|y)$$

$p(y) \to \pi_y$

$\Sigma_c)$

$x|\mu_c, \Sigma_c)$

Matrix

$$p(x|_{y=1}, \mu_2, \Sigma_2) = \frac{1}{(2\pi)^{d/2}\,\Sigma_2^{-1/2}} \exp\left(-\frac{1}{2}(x-\mu_2)^T\Sigma_2^{-1}(x-\mu_2)\right)$$

$$\log(p(y_1|x)) = \log(p(y)\,p(x|y))$$

$$= \log(\underbrace{p(y)}_{\pi_y}) + \log p(x|y)$$

$$\log(\pi_{y=1}) - \frac{1}{2}\log(\quad) - \frac{1}{2}(x-\mu_{y=1})^T\Sigma_{y=1}^{-1}(x-\mu_{y=1})$$

quadratic discriminant analysis (QDA)

$\Sigma_1 = \Sigma_2 = \Sigma$

$y \sim$ Bernoulli(

$$-\frac{1}{2}(x-\mu_{y=1})^T \Sigma_{y=1}^{-1} (x-\mu_{y=1})$$

quadratic discriminant analysis (QDA)

$\Sigma_1 = \Sigma_2 = \Sigma \qquad \mu_1, \mu_2$

$y \sim \text{Bernoulli}(\phi) \qquad p(x|y=0) \sim \mathcal{N}(\mu_0, \Sigma)$

$p(x|y=1) \sim \mathcal{N}(\mu_1, \Sigma)$