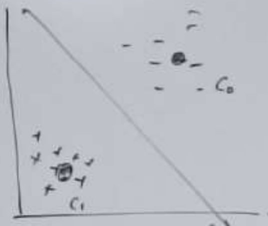
Linear Discriminant Analysis (LDA)



(I) Betw^n class separability $\uparrow$ — S_B

(II) In-class separability $\downarrow$ — S_w

C_0 $\quad$ C_1 $\quad$ $n = n_0 + n_1$

n_0 = # of points in C_0 $\quad$ n_1 = # of points in C_1

mean: $\mu_0 = \frac{1}{n_0} \sum_{x_i \in C_0} x_i$ $\quad$ $\mu_1 = \frac{1}{n_1} \sum_{x_j \in C_1} x_j$

Projected mean: $m_0 = w^T \mu_0$ $\quad$ $m_1 = w^T \mu_1$

$\sigma_0^2 = \frac{1}{n_0} \sum_{x_i \in C_0} (w^T x_i - m_0)^2$ $\quad$ $\sigma_1^2 = \frac{1}{n_1} \sum_{x_j \in C_1} (w^T x_j - m_1)^2$

$w^T x + w_0 = w^T x$

Cost fn: $J(w) = \frac{(m_0 - m_1)^2}{\sigma_0^2 + \sigma_1^2}$

$$(m_0 - m_1)^2 = (w^T \mu_0 - w^T \mu_1)^2$$
$$= w^T \left[\underbrace{(\mu_0 - \mu_1)(\mu_0 - \mu_1)^T}_{S_B = \text{Scatter matrix}} \right] w$$
$$= w^T S_B w$$

$$\sigma_0^2 = \sum (w^T x_i - m_0)^2 = \sum (w^T x_i - w^T \mu_0)^2$$
$$= w^T \left[\sum (x_i - \mu_0)(x_i - \mu_0)^T \right] w$$
$$= w^T (\quad) w$$

$+$

$$\sigma_1^2 = w^T (\quad) w$$

$$\sigma_0^2 + \sigma_1^2 = w^T \Big[\underbrace{\sum_{x_i \in C_0} (\quad) + \sum_{x_j \in C_1} (\quad)}_{S_w} \Big] w$$
$$= w^T S_w w$$

$$J(w) = \frac{w^T S_B w}{w^T S_w w}$$

$$\frac{\partial J}{\partial w} = \frac{2 S_B w}{w^T S_w w} - \ldots$$

$\Rightarrow S_w w = \ldots$

$\Rightarrow w = \lambda \ldots$

$\Rightarrow w \times \frac{1}{\lambda} = S w$

$m_1)^2$

$)^2$

$M_0)^T \big] w$

$$J(w) = \frac{w^T S_B w}{w^T S_W w}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2} = \frac{\frac{du}{dx}}{v} - \frac{u}{v^2}\frac{dv}{dx}$$

$$\frac{\partial J}{\partial w} = \frac{2 S_B w}{w^T S_W w} - \frac{(w^T S_B w)}{(w^T S_W w)^2}(2 S_W w) = 0$$

$$\Rightarrow S_W w = (\lambda) S_B w$$

$$\Rightarrow w = \lambda \underbrace{S_W^{-1} S_B}_{S} w$$

$$\Rightarrow w \times \frac{1}{\lambda} = S w$$

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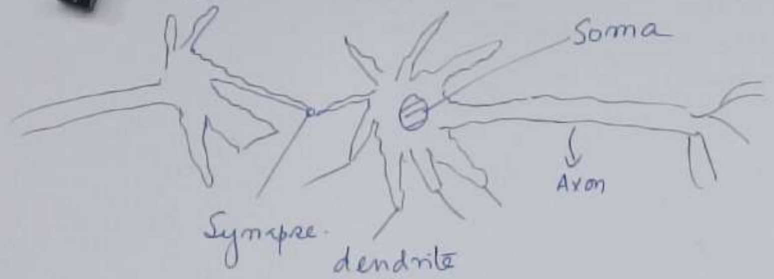
Biological Neuron



1957–1985 – Frank Rosenblatt – Perceptron

1943–44 – McCulloch + Pitts (MP model)

1965–68 – MLP

1969 – Limitation of Percep^n.

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1986 → Backprop

1989 →

$$J(w) = \frac{w^T S_B w}{w^T S_W w}$$

$$\frac{\partial J}{\partial w} = \frac{2 S_B w}{w^T S_W w} - \frac{(w^T S_B w \ldots}{(w^T S \ldots}$$

$\Rightarrow S_W w = \lambda \ldots$

$\Rightarrow w = \lambda S_W^{-1} S_B \ldots$

$\Rightarrow w \times \frac{1}{\lambda} = S w$