Quiz-1

ACOL 216: Introduction to Computer Architecture

IIT Delhi - Abu Dhabi · Semester II, 2025-26

Total Marks: 20 **Time: 40 minutes**

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1. Consider the equality $(43)_x = (y3)_8$ where x and y are unknown. What are the possible solutions of (x, y)? **(2 marks)**

   Solution:
   Expand the positional notation for both sides of the equation:

   $$4 \cdot x^1 + 3 \cdot x^0 = y \cdot 8^1 + 3 \cdot 8^0$$

   $$4x + 3 = 8y + 3$$

   Subtract 3 from both sides:

   $$4x = 8y$$

   $$x = 2y$$

   Constraints on x and y:

   - For base x, the digit 4 must be valid, so $x > 4 \implies x \geq 5$.
   - For base 8, the digit y must be valid, so $0 \leq y < 8$.
   - Since y is a leading digit in $(y3)_8$, typically $y \neq 0$.

   Substitute integer values for $y \in \{1, 2, ..., 7\}$ to find x:

   - If $y = 1 \implies x = 2$ (Invalid, x must be ≥ 5)
   - If $y = 2 \implies x = 4$ (Invalid, x must be ≥ 5)
   - If $y = 3 \implies x = 6$ (Valid)
   - If $y = 4 \implies x = 8$ (Valid)
   - If $y = 5 \implies x = 10$ (Valid)
   - If $y = 6 \implies x = 12$ (Valid)
   - If $y = 7 \implies x = 14$ (Valid)

   Possible solutions (x, y):

   $$(6, 3), (8, 4), (10, 5), (12, 6), (14, 7)$$

2. Consider the IEEE-754 single precision floating point numbers P=0xC1800000 and Q=0x3F5C2EF4. What is the product of these numbers (i.e., P × Q), represented in the IEEE-754 single precision format? **(3 marks)**

   Solution:
   First, analyze $P = \text{0xC1800000}$:

   - Binary: 1100 0001 1000 0000 ...
   - Sign (S_P): 1 (Negative)
   - Exponent (E_P): $10000011_2 = 131$. Actual: $131 - 127 = 4$.
   - Mantissa (M_P): $1.000\ldots$ (Value is 1.0).
   - Value: $-1.0 \times 2^4 = -16$.

   Analyze $Q = \text{0x3F5C2EF4}$:

   - Binary: 0011 1111 0101 1100 ...
   - Sign (S_Q): 0 (Positive)
   - Exponent (E_Q): $01111110_2 = 126$. Actual: $126 - 127 = -1$.
   - Mantissa (M_Q): $1.1011100\ldots$ (Bits from $5C2EF4$).

   Calculate Product $R = P \times Q$:

   - **Sign:** $S_R = S_P \oplus S_Q = 1 \oplus 0 = 1$ (Negative).
   - **Exponent:** $E_{actual} = E_{P,actual} + E_{Q,actual} = 4 + (-1) = 3$.
     Biased Exponent $= 3 + 127 = 130 = 10000010_2$.

- **Mantissa:** $M_R = M_P \times M_Q = 1.0 \times M_Q = M_Q$.
- The mantissa bits are identical to Q's mantissa bits (10111000010111011110100).

Construct the Result in Binary:

- Sign: 1
- Exponent: 10000010
- Fraction: 10111000010111011110100

Combine: 1 10000010 10111000010111011110100
Group into Hex:

$$\underbrace{1100}_{C}\underbrace{0001}_{1}\underbrace{0101}_{5}\underbrace{1100}_{C}\underbrace{0010}_{2}\underbrace{1110}_{E}\underbrace{1111}_{F}\underbrace{0100}_{4}$$

Final Answer: 0xC15C2EF4

3. Prove each of the following Boolean expression using Boolean algebraic laws. **($2 \times 2 = 4$ marks)**

   (i) $\overline{(A+\bar{B}+\bar{D})(C+D)(\bar{A}+C+D)(A+B+\bar{D})} \;=\; \bar{A}D+\bar{C}\bar{D}$

   (ii) $(A+C)(\bar{A}+B) \;=\; AB+\bar{A}C$

(i) $\overline{(A+\bar{B}+\bar{D})(C+D)(\bar{A}+C+D)(A+B+\bar{D})} = \bar{A}D+\bar{C}\bar{D}$
Apply De Morgan's Law (complement of product is sum of complements):

$$= \overline{(A+\bar{B}+\bar{D})} + \overline{(C+D)} + \overline{(\bar{A}+C+D)} + \overline{(A+B+\bar{D})}$$

Apply De Morgan's to individual terms:

$$= (\bar{A}BD) + (\bar{C}\bar{D}) + (A\bar{C}\bar{D}) + (\bar{A}\bar{B}D)$$

Group terms containing $\bar{A}D$:

$$= \bar{A}D(B+\bar{B}) + \bar{C}\bar{D}(1+A)$$

Since $B+\bar{B}=1$ and $1+A=1$:

$$= \bar{A}D+\bar{C}\bar{D} \quad \text{(Proven)}$$

(ii) Start with the Left Hand Side (LHS):

$$\text{LHS} = (A+C)(\bar{A}+B)$$

Expand using the Distributive Law:

$$= A\bar{A} + AB + C\bar{A} + CB$$

Apply the Inverse Law ($A\bar{A}=0$):

$$= 0 + AB + \bar{A}C + BC$$
$$= AB + \bar{A}C + BC$$

Multiply BC by 1 (Identity Law) and substitute $1 = A+\bar{A}$ (Inverse Law):

$$= AB + \bar{A}C + BC(A+\bar{A})$$

Distribute BC into the parenthesis:

$$= AB + \bar{A}C + ABC + \bar{A}BC$$

Group terms containing AB and terms containing $\bar{A}C$:

$$= (AB + ABC) + (\bar{A}C + \bar{A}BC)$$

Factor out AB from the first group and $\bar{A}C$ from the second:

$$= AB(1+C) + \bar{A}C(1+B)$$

Apply the Annulment Law ($1+X=1$):

$$= AB(1) + \bar{A}C(1)$$

Apply the Identity Law ($X \cdot 1 = X$):

$$= AB + \bar{A}C \quad \text{(Proven)}$$

4. Perform the following subtraction operations using the r**'s complement method**. For each case, explicitly show the calculation of the complement for the subtrahend and the subsequent addition step.

   (3+4 = 7 marks)

   (i) $(101.101)_2 - (11.011)_2$ [Use 2's complement; Express your final answer in **base-2**]

(ii) $(256.34)_7 - (143.56)_7$ [Use 7's complement; Express your final answer in **base-7**]

(i) $(101.101)_2 - (11.011)_2$ using 2's complement.
Minuend $M = 101.101$
Subtrahend $S = 011.011$ (padded with leading zero)
Step 1: Find 2's complement of S.

- 1's complement of 011.011 is 100.100.
- Add 1 to LSB: $100.100 + 0.001 = 100.101$.

Step 2: Add M + 2's Comp(S).

$$\begin{array}{rrl} & 101.101 & \\ + & 100.101 & \\ \hline & 010.010 & \text{(Carry out 1)} \end{array}$$

Result includes a carry out of 1, which indicates the result is positive. Discard the carry.
Result $= 010.010_2 = \mathbf{10.01_2}$

(ii) $(256.34)_7 - (143.56)_7$ using 7's complement.
Minuend $M = 256.34$
Subtrahend $S = 143.56$
Step 1: Find 7's complement of S.

- 6's complement (subtract digits from 6):
  $666.66 - 143.56 = 523.10$
- Add 1 to LSB (7^{-2}):
  $523.10 + 0.01 = 523.11$

Step 2: Add M + 7's Comp(S) in Base 7.

$$\begin{array}{rr} & 256.34_7 \\ + & 523.11_7 \\ \hline \end{array}$$

Addition steps:

- $4 + 1 = 5$
- $3 + 1 = 4$
- $6 + 3 = 9 \to 12_7$ (Write 2, Carry 1)
- $5 + 2 + 1 = 8 \to 11_7$ (Write 1, Carry 1)
- $2 + 5 + 1 = 8 \to 11_7$ (Write 1, Carry 1)

Sum: 1112.45_7. Discard the carry out.
Final Answer: $\mathbf{112.45_7}$

5. What is the base of the number system in which the following equation holds: $\frac{312}{20} = 13.1$? **(2 marks)**

Solution:
Let the base be b. Convert the equation to polynomial form:

$$\frac{3b^2 + 1b + 2}{2b + 0} = 1b + 3 + \frac{1}{b}$$

Multiply both sides by $2b$:

$$3b^2 + b + 2 = 2b(b + 3 + b^{-1})$$
$$3b^2 + b + 2 = 2b^2 + 6b + 2$$

Rearrange to form a quadratic equation:

$$3b^2 - 2b^2 + b - 6b + 2 - 2 = 0$$
$$b^2 - 5b = 0$$
$$b(b - 5) = 0$$

Possible solutions are $b = 0$ or $b = 5$. Since a base cannot be 0 and digits like 3 exist (requiring $b > 3$), the only valid solution is:
Base = 5

6. Consider a hypothetical ISA where each instruction is exactly 4 bytes long. Conditional and unconditional branch instructions in this ISA use PC-relative addressing mode with *Offset* specified in bytes to the target location of the branch instruction. Further, the *Offset* is always with respect to the address of the next instruction in the program sequence. Also, the `add` and `sub` instructions follow similar definitions and syntax as SimpleRISC, while the `cmp` and `beq` instructions are defined as:

- `cmp R1, R2, R3`: This instruction compares the values stored in registers `R2` and `R3`. The result of this comparison is stored in register `R1`, which acts as a condition register for subsequent control flow.
- `beq R1, Offset`: This is a Branch if Equal instruction. It examines the comparison result stored in register `R1`; if the condition for equality is met, the program execution jumps to the target address calculated using PC-relative addressing with the *Offset* specified in the instruction.

Now consider the following instruction sequence:

Instr. No.	**Instruction**
i:	`add R2, R3, R4`
$i+1$:	`sub R5, R6, R7`
$i+2$:	`cmp R1, R9, R10`
$i+3$:	`beq R1, Offset`

If the target of the branch instruction is i, **then what is the decimal value of the *Offset*?** **(2 marks)**

Solution:

The instruction set uses PC-relative addressing where the Offset is relative to the *next* instruction.

- Instruction size = 4 bytes.
- Branch instruction location: Index $i+3$.
- Next instruction location (PC_{next}): Index $i+4$.
- Target location (Instruction i): Index i.

The target address equation is:

$$\text{Target\_Addr} = \text{PC}_{next} + \text{Offset}$$

$$\text{Address}(i) = \text{Address}(i+4) + \text{Offset}$$

Convert indices to bytes (multiply by 4):

$$4i = 4(i+4) + \text{Offset}$$

$$4i = 4i + 16 + \text{Offset}$$

$$\text{Offset} = -16$$

Decimal Value of Offset: -16